COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1945

VOLUME XLVIII

№ 6

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

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NOUVELLE SÉRIE

13-me année

Paraissant tous les dix jours

1945

VOLUME XLVIII, № 6

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Édition de l'Académie des Sciences de l'URSS
Moscou 1945

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MATHÉMATIQUES

LA GÉOMÉTRIE DU CHAMP DE VECTEURS. II

Par S. S. BUSCHEGUENNCE

(Présenté par N. N. Lusin, de l'Académie, le 17.VII.1944)

Dans ce qui suit je retiens toutes les conceptions et les désignations de ma Note précédente (1).

1. Supposons que la courbure totale du champ (J_3) sera nulle et prenons pour J_1 la direction d'une ligne de courbure du champ, alors on obtient

$$p_1 = 0, \quad p_2 q_1 = 0$$

deux cas possibles de ces conditions sont équivalents et ne diffèrent que par le choix d'un repère. Tous deux donnent: une direction de courbure est orthogonale à l'une des directions asymptotiques, l'autre direction de courbure confond à la seconde direction asymptotique; cette propriété caractérise le champ de vecteurs à condition que $K_t = 0$.

2. Supposons que les lignes de courbure du champ sont indéterminées, alors

$$p_1 = 0, \quad p_2 + q_1 = 0, \quad q_2 = 0$$

et puisque la condition 7 (1) est remplie, le champ admet une famille de surfaces orthogonales. Les lignes de courbure sur ces surfaces sont les lignes de courbure du champ, donc cette famille de surfaces consiste des sphères si $K_t \neq 0$ ou des plans si $K_t = 0$.

3. Si les asymptotiques du champ sont indéterminées, nous avons

$$q_1 = 0, \quad q_2 - p_1 = 0, \quad p_2 = 0$$

Alors si de plus $K_t = 0$, le champ aura une famille de surfaces orthogonale consistant des plans. Si $K \neq 0$, le champ de vecteurs n'admet pas de famille orthogonale, et se laisse obtenir par la construction suivante. Prenons une famille des cylindres circulaires ayant le même axe; pour un point $\overline{M}$ quelconque de l'espace nous devons prendre le vecteur J_3 du champ dans le plan tangent au cylindre correspondant (à travers le point $\overline{M}$) à l'angle $\varphi = \operatorname{arctg} \frac{R}{C}$ avec la génératrice rectiligne, où R est le rayon du cylindre et C une constante arbitraire.

4. Analogiquement à la conception des surfaces de rotation je vais dénommer un champ de vecteurs, champ de rotation, si tous ses vecteurs coupent un axe constant.

Dans un champ de rotation les vecteurs perpendiculaires aux vecteurs du champ et coupants l'axe fixe enveloppent une famille des lignes de courbure.

Je propose d'appeler les lignes de cette famille «méridiennes du champ de rotation».

Pour le déplacement le long d'un vecteur J_3 du champ de rotation le vecteur de Darboux doit être perpendiculaire à la méridienne. Enfin pour un déplacement arbitraire dans un champ de rotation le vecteur de Darboux doit être dans un plan perpendiculaire au plan $(\bar{c}, J_3)$ ou au plan (J_3, J_1), où $\bar{c}$ est l'axe fixe et J_1 est une direction des lignes de courbure du champ.

5. J'appelle un champ de vecteurs champ du complexe spécial, s'il appartient à un complexe spécial, c'est-à-dire, à un complexe des tangents d'une surface. Pour qu'un champ de vecteurs appartienne à un complexe spécial, il faut et il suffit que dans chaque point pour le déplacement le long du vecteur du champ le vecteur de Dardoux soit toujours perpendiculaire à la direction de la ligne de courbure d'une famille.

6. Dans le choix du repère pour une congruence de courbes il importe de discuter la question, s'il est possible de choisir deux familles orthogonales de surfaces à travers des courbes de la congruence donnée.

J'affirme que si une congruence donnée des courbes admet deux familles de surfaces orthogonales, l'un champ des bissectrices des asymptotiques doit admettre une famille de surfaces orthogonales et celle-ci sera l'une des familles cherchées.

7. En conservant toujours J_3 comme vecteur du champ, prenons au lieu du repère $J_1J_2J_3$ un nouveau repère

$$\tilde{J}_1 = J_1 \cos\sigma + J_2 \sin\sigma,\ \tilde{J}_2 = -J_1 \sin\sigma + J_2 \cos\sigma,\ \tilde{J}_3 = J_3$$

alors les nouvelles formes correspondantes seront

$$\tilde{\omega}_0^1 = \omega_0^1 \cos\sigma + \omega_0^2 \sin\sigma \qquad \tilde{p} = p\cos\sigma + q\sin\sigma$$

$$\tilde{\omega}_0^2 = -\omega_0^1 \sin\sigma + \omega_0^2 \cos\sigma \qquad \tilde{q} = -p\sin\sigma + q\cos\sigma$$

$$\tilde{\omega}_0^3 = \omega_0^3 \qquad \tilde{r} = r + d\sigma$$

Ainsi les coefficients de formes p et q donnent lieu à cinq invariants. Nous pouvons chercher ces invariants parmi les invariants communs des formes quadratiques

$$(d\bar{M})^2,\ d\bar{M}J_3 dJ_3,\ d\bar{M}dJ_3$$

Nous recevons six invariants, précisément

$$i_1 = p_1 + q_2, \qquad i_2 = p_2 - q_1,\ i_3 = p_1q_2 - p_2q_1,\ i_4 = p_3^2 + q_3^2$$

$$i_5 = p_1q_3^2 - (p_2 + q_1)\,q_3p_3 + q_2p_3^2,\ \ i_6 = p_2q_3^2 + (p_1 - q_2)\,q_3p_3 - q_1p_3^2$$

tous ces invariants sont liés par une relation

$$i_3i_4^2 = i_1i_4i_5 + i_2i_4i_6 - i_5^2 - i_6^2$$

L'annulation de i_1 signifie que les lignes de courbure du champ sont orthogonales ou que le champ admet une famille de surfaces orthogonales. L'inva-

riant i_2 est la courbure moyenne du champ; l'annulation de i_2 signifie que les asymptotiques du champ sont orthogonales.

L'invariant i_3 est la courbure totale du champ.

L'invariant i_4 est le carré de la projection $\overline{\omega}_0$ (vecteur de Darboux pour le déplacement le long du vecteur du champ) sur le plan orthogonal an vecteur du champ.

L'annulation i_5 est un symptôme que la forme quadratique $d\overline{M}J_3dJ_3$ se démembre en deux facteurs linéaires; on peut aussi dire que cette circonstance signifie que la direction perpendiculaire à $\overline{\omega}_0$ (bien entendu, dans le plan normal à J_3) donne l'une direction des lignes de courbure.

Enfin, l'annulation de i_6 est une marque que la forme quadratique $d\overline{M}dJ_3$ se démembre en deux facteurs linéaires, qui signifie en même temps que la direction perpendiculaire à $\overline{\omega}_0$ (dans le plan orthogonal à J_3) est l'une des directions asymptotiques.

L'Institut de Hydrotechnique
et d'Amélioration.
Moscou.

Manuscrit reçu
le 17. VII. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MATHEMATICS

GEOMETRY OF FIELD OF LOCAL CENTRAL PLANE CURVES IN X_3

By V. WAGNER

(Communicated by A. N. Kolmogoroff, Member of the Academy, 31.V.1945)

In the previous paper (1) considering geometry of field of local curves in X_3 we have excluded the case, where the local curves are plane curves whose plane passes through the centre of the tangent E_3. Now we consider this exceptional case separately because the general theory developed in the previous paper cannot be applied without essential modifications. Let

$$x^\alpha = l^\alpha(\xi^\lambda, \theta) \tag{1}$$

be the equations of a field of local central plane curves in X_3, where the parameter θ is the centro-affine length of arc of the curves. Thus, the permissible transformations of the parameter are

$$'\theta = \theta + C(\xi^\lambda) \tag{2}$$

where $C(\xi^\lambda)$ is an arbitrary function of ξ^λ.

The vector $m^\alpha = \partial l^\alpha / \partial \theta$ is the unit tangent vector of the curves.

We have

$$\frac{\partial m^\alpha}{\partial \theta} = -\varepsilon l^\alpha - \varkappa m^\alpha \tag{3}$$

where $\varepsilon = \pm 1$ and $\varkappa$ is the centro-affine curvature of the curves. Each curve of the field is lying in a central plane in the corresponding tangent E_3, whose equation is

$$\pi_\alpha(\xi^\lambda)\, x^\alpha = 0 \tag{4}$$

where π_α is a covariant pseudo-vector in E_3, *i. e.* vector given within an arbitrary factor. Thus, we obtain a field of local central planes in X_3. This field is said to be holonomic if the planes are tangent to a certain parameter family of surfaces in X_3, and non-holonomic in the opposite case. In the holonomic case the theory of field of local central curves can be reduced to the theory of field of local curves in X_2 which is equivalent to the geometry of two-dimensional Finslerian space, if the curves of the field are taken as indicatrices of Minkovskian metric in the tangent E_2. From the point of view of application to the calculus of variations the holonomic case is not interesting because the corresponding problem of Lagrange can easily be reduced to the ordinary problem of calculus of variations. Let us suppose that the field of

local central planes is non-holonomic. In this case $l^\alpha m^\beta \partial_{[\alpha} n_{\beta]} \neq 0$, and at each point of local curve we can define a covariant vector $n_\lambda = \dfrac{n_\lambda}{2l^\alpha m^\mu \partial_{[\lambda} n_{\mu]}}$. By means of the equation

$$6E_{\alpha\beta\gamma} l^\alpha m^\beta = n_\gamma \tag{5}$$

we define a skew symmetric affinor $E_{\alpha\beta\gamma}$ and introduce two covariant bivectors

$$L_{\alpha\beta} = 3E_{\alpha\beta\gamma} l^\gamma; \quad M_{\alpha\beta} = 3E_{\alpha\beta\gamma} m^\gamma. \tag{6}$$

Let us introduce the differential forms

$$n = n_\alpha d\xi_\alpha; \quad L = L_{\alpha\beta} [d\xi^\alpha d\xi^\beta]; \quad M = M_{\alpha\beta} [d\xi^\alpha d\xi^\beta]; \quad E = E_{\alpha\beta\gamma} [d\xi^\alpha d\xi^\beta d\xi_\gamma]$$

where the brackets denote the Cartan skew symmetric multiplication. Let us suppose that the local curves of considered field satisfy the condition

$$2\varkappa^2 + \varepsilon \neq 0 \tag{7}$$

which means that we exclude from our consideration the curves whose equation, with respect to a suitably chosen system of coordinates in plane, is $(x^1)^2 x^2 = 1$. If the condition (7) holds, an invariant linear connexion in the compound manifold $x_{3+(1)}$ corresponding to the field of local curves can be defined by the following relations

$$(DL) = 0; \quad (DM) = 0; \quad (nR) = 0 \tag{8}$$

where D is the symbol of the absolute differential and R is the curvature form of connexion.

Let

$$\delta\theta = d\theta + \Gamma = 0 \tag{9}$$

where $\Gamma = \Gamma_\alpha (\xi^\lambda, \theta) d\xi^\alpha$ is the equation determining the invariant connexion. Denoting $\lambda = [\partial L]/E$; $\mu = [\partial M]/E$, we obtain from (8)

$$[\Gamma L] = \gamma_l E; \quad [\Gamma M] = \gamma_m E;$$

$$[\Gamma \partial n] = \left\{ \partial_m \gamma_l - \partial_l \gamma_m + \frac{\partial \gamma_m}{\partial \theta} \gamma_l - \frac{\partial \gamma_l}{\partial \theta} \gamma_m - \gamma_l \mu - \gamma_m \lambda + \right.$$

$$\left. + \varepsilon \gamma_l^2 + \gamma_m^2 + \varkappa \gamma_l \gamma_m \right\} E \tag{10}$$

where

$$\gamma_l = \frac{\varkappa\lambda - \mu}{2\varkappa + \varepsilon}; \quad \gamma_m = \frac{\varepsilon\lambda + 2\varkappa\mu}{2\varkappa^2 + \varepsilon};$$

$$\partial_l = l^\alpha \frac{\partial}{\partial \xi^\alpha}; \quad \partial_m = m^\alpha \frac{\partial}{\partial \xi^\alpha}; \quad \partial = d\xi^\alpha \frac{\partial}{\partial \xi^\alpha} \tag{11}$$

Since the bilinear differential forms L, M and $[\partial n]$ are independent, the equations (10) determine uniquely the Pfaffian of connexion Γ. By virtue of the relation $[nDn] = [n\partial n] = E$ the bilinear forms L, M, $[Dn]$ are also independent, and the equations

$$[lL] = E; \quad [lM] = 0; \quad [lDn] = 0; \quad [mL] = 0; \quad [mM] = E; \quad [mDn] = 0 \tag{12}$$

uniquely determine two Pfaffians l and m invariantly connected with the field of local central curves. Consider the contravariant vectors reciprocal to the covariant vectors corresponding to the Pfaffians l, m, n. It is easy to see that two of these contravariant vectors coincide with the vectors l^α and m^α. The third vector will be denoted by n^α and called a binormal vector of the local curves.

We have

$$\frac{\partial l}{\partial \theta} = \varepsilon m + pn; \quad \frac{\partial m}{\partial \theta} = -l + \varkappa m + qn; \quad \frac{\partial n}{\partial \theta} = \varkappa n \tag{13}$$

where

$$p = \varepsilon \frac{2\varkappa D_l \varkappa - D_m \varkappa}{2\varkappa^2 + \varepsilon}; \quad q = \frac{\varepsilon D_l \varkappa + \varkappa D_m \varkappa}{2\varkappa^2 + \varepsilon} \tag{14}$$

the differential operators D_l, D_m, D_n being defined by the symbolical relation $D = lD_l + mD_m + nD_n$.

It follows from (13) that

$$\frac{\partial n^\alpha}{\partial \theta} = -pl^\alpha - qm^\alpha - \varkappa n^\alpha \tag{15}$$

Thus, a necessary and sufficient condition that the invariant binormal vector n^α be constant along each local curve is that $\varkappa = 0$. In this case we have a field of local conics whose centres coincide with the centres of the tangent E_3. Taken as the metric indicatrices these conics determine Euclidean metric in the planes where they are lying. It follows that the geometry of field of local central conics in X_3 is identical with the inner geometry of metric two-dimensional non-holonomic manifold V_3^2 in X_3. The constant binormal vector n^α of the local conics determines an invariant rigging of this V_3^2. The extended parallel displacement of admissible vectors in V_3^2 (2), corresponding to this rigging, coincides with the parallel displacement of admissible vectors determined by the connexion (9) in $X_{3+(1)}$.

The problem of equivalence of two fields of local central plane curves in X_3 requires the finding of complete set of differential invariants. It can be shown that the complete set of differential invariants for a field of local central plane curves can be obtained by applying the differential operators D_l, D_m, D_n, $\partial/\partial\theta$ to the invariants

$$\varkappa; \; H = \frac{[mDl]}{E}; \quad P = \frac{[mDn]}{E} \tag{16}$$

A field of local central plane curves corresponds to a problem of Lagrange which being given in the ordinary way has a non-integrable Pfaffian equation in the variables ξ^α as the supplementary condition. Let ${}^v d\xi^\alpha$ be admissible variations of an admissible curve, *i. e.* such variations where the varied curve is also admissible. Denoting $l_v - l_\alpha {}^v d\xi^\alpha$, $m_v = m \; {}^v d\xi^\alpha$, $n_v = n_\alpha {}^v d\xi^\alpha$ we obtain that they satisfy the single condition

$$\frac{dn_v}{ds} - \varkappa \frac{\delta\theta}{ds} n_v - m_v = 0 \tag{17}$$

By means of this equation the variation of the length of arc can be represented thus

$${}^v ds = \left[l_v - \varepsilon \frac{\delta\theta}{ds} n_v \right]_{s_1}^{s_2} + \int_{s_1}^{s_2} \left\{ \varepsilon \frac{d}{ds}\left(\frac{\delta\theta}{ds}\right) + \varepsilon\varkappa \left(\frac{\delta\theta}{ds}\right)^2 - p\frac{\delta\theta}{ds} + H \right\} n_v \, ds \tag{18}$$

whence the differential equations of the extremals are

$$\frac{d\xi^\alpha}{ds} = l^\alpha(\xi^\lambda, \theta); \; \varepsilon \frac{d}{ds}\left(\frac{\delta\theta}{ds}\right) + \varepsilon\varkappa \left(\frac{\delta\theta}{ds}\right)^2 - p\frac{\delta\theta}{ds} + H = 0 \tag{19}$$

With each point of extremal is associated the transversal plane in the tangent E_3 which passes through the tangent of the indicatrix. The transversal plane is determined by the covariant vector $e_\alpha = l_\alpha - \varepsilon \frac{\delta\theta}{ds} n_\alpha$. The geometrical meaning of the Weierstrasse condition is the same as for the ordinary problem of calculus of variations, *i. e.* the indicatrix must be convex.

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of the Leningrad University.

Received
3.IV.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MATHÉMATIQUES

SUR QUELQUES SYSTÈMES NON ORTHOGONAUX

Par V. STEPANOFF

(Présenté par A. N. Kolmogoroff, de l'Académie, le 15.XI.1944)

Pour résoudre par la méthode de Ritz certains problèmes de la théorie de l'élasticité, M. Filonenko-Boroditch a considéré le système des fonctions.

$$1-\cos 2x,\ \cos 2x-\cos 4x,\ \ldots,\ \cos x-\cos 3x,\ \cos 3x-\cos 5x,\ \ldots,\ 0\leqslant x\leqslant\pi$$

Il a démontré que ce système est fermé. Dans cette Note je donne des conditions nécessaires et suffisantes pour qu'un système d'une forme assez générale, composé de formes linéaires de fonctions d'un système orthogonal arbitraire, soit complet dans l'espace du système orthogonal.

Soit $\{\varphi_n\}$ un système orthogonal et normal de fonctions réelles à carré sommable, il définit l'espace linéaire $\mathfrak{H}_2$. Nous considérons un système (non orthogonal)

$$\psi_i=a_{i1}\varphi_1+a_{i2}\varphi_2+\ldots+a_{in_i}\varphi_{n_i};\quad i=1,\ 2,\ldots, \tag{1}$$

a_{i1} étant des constantes réelles, $a_{in_i}\neq 0$, n_i ne décroît pas pour i croissant; le système (1) vérifie d'ailleurs les deux conditions suivantes:

I. Les fonctions ψ_i sont linéairement indépendantes.

II. On a $\liminf\limits_{i\to\infty}(n_i-i)=m\quad 0\leqslant m<\infty$.

On cherche des conditions pour que le système (1) soit complet. Nous substituons d'abord au système (1) un système équivalent réduit à une forme canonique.

Il résulte de la condition II qu'il existe une suite infinie de valeurs croîssantes de i, $i_0<i_1<\ldots<i_k<\ldots$, telles que $n_{ik}-i_k=m$, $k=0,\ 1,\ 2,\ \ldots$

Partageons la suite (1) en collections contenant successivement i_0, i_1-i_0, $\ldots$, i_k-i_{k-1}, $\ldots$ fonctions. La collection de rang k, $k\geqslant 1$, contient i_k-i_{k-1} fonctions ψ_i et le même nombre de fonctions nouvelles φ_i (n'appartenant pas aux collections précédentes). Désignons par $\mathbf{M}_k$ la matrice des coefficients a_{ij}, $i_{k-1}<i\leqslant i_k$, $i_{k-1}+m<j\leqslant i_k+m$; et par D_k son déterminant. Deux cas sont à considérer.

1. Il n'existe qu'un nombre fini de D_k qui sont nuls. En augmentant s'il est nécessaire i_0, nous pouvons supposer $D_k\neq 0$, $k=1,\ 2,\ \ldots$ En vertu de la condition I les équations (1) de la collection de rang 0 peuvent être résolues par rapport à i_0 des fonctions φ_i que nous désignerons en changeant les notations par $\varphi^{(1)}$, $\varphi^{(2)}$, $\ldots$, $\varphi^{(i_0)}$; ces fonctions s'exprimeront linéairement par les m fonctions φ_i restées, que nous désignerons par $\chi_1,\ \chi_2,\ \ldots,\ \chi_m$, ainsi que par i_0 formes linéaires indépendantes des fonctions ψ_i; en considérant ces formes comme fonctions $\overline{\psi}_i$ nouvelles, nous avons ainsi réduit la collection de rang 0 à la forme canonique

$$\overline{\psi}_i=\varphi^{(i)}+\sum_{j=1}^{m}k_j^{(i)}\chi_j,\ i=1,\ 2,\ \ldots,\ i_0. \tag{2}$$

Admettons que $k-1$ collections sont déjà réduites à la forme (2). En vertu de la condition $D_k\neq 0$ les fonctions $\varphi_{i_{k-1}+m+1}$, $\ldots$, φ_{i_k+m} de la k-ième collection (que nous désignerons par $\varphi^{(i_{k-1}+1)}$, $\ldots$, $\varphi^{(i_k)}$) peuvent être exprimés

à l'aide des $\varphi^{(i)}$, $i \leqslant i_{k-1}$, des χ_j, $j=1, 2, \ldots, m$, et de certaines formes linéaires indépendantes des ψ_i, $i_{k-1} < i \leqslant i_k$; ensuite nous éliminons à l'aide des expressions (2) déjà obtenues, les fonctions $\varphi^{(i)}$, $i \leqslant i_{k-1}$ et, nous obtenons de nouvelles fonctions $\bar{\psi}$, $i_{k-1} < i \leqslant i_k$ formes linéaires distinctes de $\psi_i, \ldots, \psi_{ik}$ exprimées par les formules de la même forme canonique (2). Chaque fonction ψ_i s'exprimant à l'aide d'un nombre fini de fonctions $\bar{\psi}$ et inversement, le système (2) infiniment prolongé est complet ou non complet en même temps que le système (1).

2. $D_k = 0$ pour une suite infinie de valeurs de k. En vertu de la condition I le rang r_k de la matrice $\mathbf{M}_k$ est inférieur à son ordre $i_k - i_{k-1}$ de m unités au plus; soit

$$\limsup_{k\to\infty} (i_k - i_{k-1} - r_k) = m - m_1, \quad 0 \leqslant m_1 < m$$

Soit $\{i_{k'}\}$ une suite partielle de nombres i_k vérifiant l'égalité $i_{k'} - i_{k'-1} - r_k = m - m_1$. Il existe $m - m_1$ formes linéaires indépendantes de la collection du rang k' ne contenant pas de fonctions φ_i pour $i_{k'-1} + m < i \leqslant i_{k'} + m$; nous donnons aux formes linéaires $\bar{\psi}$ correspondantes les $m - m_1$ premières places dans cette collection, tandis que les autres formes linéaires indépendantes $\bar{\psi}$ dont le nombre est $i_{k'} - i_{k'-1} - (m - m_1)$, occupent les places suivantes. On obtient une suite illimitée de nombres croissants $l_{k-1} = i_{k'-1} + m - m_1$; pour chacun de ces nombres la différence entre le nombre des fonctions φ et celui des ψ a la valeur $i_{k'-1} + m - [i_{k'-1} + (m - m_1)] = m_1$, donc nous avons réduit le système (1) à un autre équivalent, vérifiant la condition II dans laquelle m est remplacé par un nombre $m_1 < m$. Après un nombre fini μ de transformations nous aboutirons à un système où le nombre m_μ ne peut pas être abaissé. Si l'on a $m_\mu = 0$, il y aura une infinité d'indices i tels que le nombre des fonctions φ est égal à celui des ψ; le système réduit de la forme (1) peut être résolu par rapport aux fonctions φ, donc le système $\{\psi_m\}$ est complet d'une façon banale. Dans tous les autres cas on obtient la forme canonique (2) où $m > 0$. En définitive, en changeant encore légèrement les notations, nous considérons le système

$$\psi_i = \varphi_i + \sum_{j=1}^{m} k_j^{(i)} \chi_j, \quad i = 1, 2, \ldots, m \geqslant 1 \tag{3}$$

$\chi_1, \chi_2, \ldots, \chi_m, \varphi_1, \varphi_2, \ldots$ formant un système orthogonal normé de fonctions réelles à carré sommable définissant un espace linéaire $\mathfrak{H}_2$ et $k_j^{(i)}$ étant des constantes réelles. Introduisons les notations:

$$\sum_{i=1}^{n} k_j^{(i)} k_l^{(i)} = (k_j, k_l)_n, \quad (k_{j_1}, k_{j_1})_n = \Delta_n(j_1),$$

$$\begin{vmatrix} (k_{j_1}, k_{j_1})_n & (k_{j_1}, k_{j_2})_n & \ldots & (k_{j_1}, k_{j_s})_n \\ \ldots & \ldots & \ldots & \ldots \\ (k_{j_s}, k_{j_1})_n & (k_{j_s}, k_{j_2})_n & \ldots & (k_{j_s}, k_{j_s})_n \end{vmatrix} = \Delta(j_1, j_2, \ldots, j_s);$$

$$s \leqslant m, \quad 1 \leqslant j_1 < j_2 < \ldots < j_s \leqslant m$$

L'égalité facile à vérifier a lieu

$$\Delta_n(j_1, j_2, \ldots, j_s) = \frac{1}{s!} \sum_{i_1, i_2, \ldots, i_s = 1}^{n} \begin{vmatrix} k_{j_1}^{(i_1)} k_{j_1}^{(i_1)} & \ldots & k_{j_s}^{(i_1)} \\ \ldots & \ldots & \ldots \\ k_{j_1}^{(i_s)} k_{j_2}^{(i_s)} & \ldots & k_{j_s}^{(i_s)} \end{vmatrix}^2$$

Il s'ensuit que n croissant Δ_n ne décroît pas, $\Delta_n(j_1, j_2, \ldots, j_s) \geqslant 0$, l'égalité n'ayant lieu que lorsqu'il existe une relation linéaire entre les vecteurs $(k_{j_l}^{(1)}, k_{j_l}^{(2)}, \ldots, k_{j_l}^{(n)})$, $l = 1, 2, \ldots, s$. Evidemment Δ_n est analogue au déterminant de Gram.

Théorème. *La condition nécessaire et suffisante pour que le système* (3) *soit complet, est*

$$\lim_{n\to\infty} \frac{\Delta_n(2, 3, \ldots, m)+\Delta_n(1, 3, \ldots, m)+\ldots+\Delta_n(1, 2, \ldots, m-1)}{\Delta_n(1, 2, \ldots, m)}=0 \qquad (A)$$

J'avais démontré ce théorème dans les termes de systèmes fermés; dans la séance de la Société Mathématique de Moscou I. Gelfand et N. Bary ont fait la remarque que le calcul est simplifié si l'on prend pour base la notion de système complet, or les deux définitions sont équivalentes dans L_2.

Démonstration. Admettons que le système (3) ne soit pas complet dans $\mathfrak{H}_2$. Alors il existe une fonction $f\in\mathfrak{H}_2$ orthogonale à toutes les ψ_i, soit

$$f\sim\sum_{j=1}^{m} b_j\chi_j+\sum_{i=1}^{\infty} a_i\varphi_i \qquad (4)$$

Les conditions d'orthogonalité $(f, \psi_i)=0$ donnent

$$b_1 k_1^{(i)}+b_2 k_2^{(i)}+\ldots+b_m k_m^{(i)}+a_i=0,\ i=1, 2, \ldots \qquad (5)$$

La série $\sum_{i=1}^{\infty} a_i^2$ étant convergente, il en résulte une condition nécessaire et suffisante pour que le système (3) ne soit pas complet: il existe des quantités réelles $b_1, b_2, \ldots, b_m$ telles que l'on a

$$\lim_{n\to\infty}\sum_{i=1}^{n}(b_1 k_1^{(i)}+\ldots+b_m k_m^{(i)})^2<\infty$$

En développant la quantité sous le signe lim nous obtenons une forme quadratique définie

$$\sum_{r,t=1}^{m}(k_r, k_t)_n\, b_r b_t \qquad (6)$$

et dans le cas $m=1$ le monôme quadratique $(k_1, k_1)_n b_1^2$. L'analyse du dernier cas est immédiate: on a forcement $b_1\neq 0$, sinon d'après (5) tous les a_i seraient nuls, donc *le système* (2) *pour* $m=1$ *est complet si* $\sum_{i=1}^{\infty} k_1^{(i)2}$ *diverge, et il est incomplet si cette série converge, la fonction orthogonale aux* ψ_i *étant dans le dernier cas* $(b_1=1)$ $f\sim\chi_1-\sum_{i=1}^{\infty} k_1^{(i)}\varphi_i$.

Pour $m>1$ la valeur minimum de la forme (6) soumise à la condition $\sum_{j=1}^{m} b_j^2=1$ est égale à la plus petite racine (nécessairement positive, ne décroissant pas quand n croît) de l'équation

$$\begin{vmatrix} (k_1, k_1)_n-\lambda & (k_1, k_2)_n & \ldots & (k_1, k_m)_n \\ \ldots & \ldots & \ldots & \ldots \\ (k_m, k_1)_n & (k_m, k_2)_n & \ldots & (k_m, k_m)_n-\lambda \end{vmatrix}=0$$

Les racines de cette équations étant $\lambda_1^{(n)}\leqslant\lambda_2^{(n)}\leqslant\ldots\leqslant\lambda_m^{(n)}$, nous obtenons

$$\frac{1}{\lambda_1^{(n)}}+\frac{1}{\lambda_2^{(n)}}+\ldots+\frac{1}{\lambda_m^{(n)}}=\frac{\sum_{t=1}^{m}\Delta_n(1, \ldots, t-1, t+1, \ldots, m)}{\Delta_n(1, 2, \ldots, m)} \qquad (7)$$

Si la condition (A) se trouve vérifiée, il résulte de (7) $\lim_{n\to\infty}\lambda_1^{(n)}=\infty$; $\lambda_1^{(n)}$ étant la valeur minimum de $\sum_{i=1}^{n} a_i^2$ pour tout choix des constantes b_j, la série $\sum_{i=1}^{\infty} a_i$ est divergente, il n'existe pas de fonction $f\in\mathfrak{H}_2$ orthogonale au système (3) qui est par conséquent complet.

Si, au contraire, la condition (A) n'est pas vérifiée, $\lambda_1^{(n)}$ est borné pour une suite infinie de valeurs de n (donc, vu la croissance de $\lambda_1^{(n)}$ il est borné par toutes les valeurs de n); soit $\lambda_1=\lim_{n\to\infty}\lambda_1^{(n)}$. Les valeurs rendant la forme (6) minimum seront désignées par $b_j^{(n)}$; comme on a $\sum_{j=1}^{m} b_j^{(n)2}=1$ il est possible de trouver une suite $\{n_k\}$ telle que $\lim_{k\to\infty} b_j^{(n_k)}=b_j$, $j=1, 2, \ldots m$, $\sum_{j=1}^{m} b_j^2=1$. Les équations (5) fournissent les valeurs correspondantes pour $a_i^{(n_k)}$ et on a

$$\sum_{i=1}^{n_k} a_i^{(n_k)2}=\lambda_{n_k},\quad \lim_{k\to\infty} a_i^{(n_k)}=a_i,\quad i=1, 2, \ldots$$

On démontre sans peine la relation $\sum_{i=1}^{\infty} a_i\leqslant\lambda_1$, et la fonction $f\in\mathfrak{H}_2$, définie par (4) est bien orthogonale au système (3) qui est par conséquent incomplet. Le théorème est démontré.

Remarques. 1. Si la condition (A) est vérifiée, on conclut à plus forte raison qu'il n'existe pas de fonctions orthogonales à $\{\psi_n\}$ de la forme

$$f\sim\sum_{l=1}^{s} b_{j_l}\chi_{j_l}+\sum_{i=1}^{\infty} a_i\varphi_i;\ 1\leqslant j_1<j_2<\ldots\leqslant j_s\leqslant m$$

par conséquent l'égalité (A) entraîne les égalités suivantes:

$$\lim_{n\to\infty}\frac{1}{\Delta_n(j_1, j_1)}=0 \qquad (A_1)$$

$$\lim_{n\to\infty}\frac{\Delta_n(j_1)+\Delta_n(j_2)}{\Delta_n(j_1, j_2)}=0 \qquad (A_2)$$

. . . . . .

$$\lim_{n\to\infty}\frac{\Delta_n(j_2, \ldots, j_{m-1})+\ldots+\Delta_n(j_1, \ldots, j_{m-2})}{\Delta_n(j_1, j_2, \ldots, j_{m-1})}=0 \qquad (A_{m-1})$$

2. Il est aisé de déterminer le nombre L de fonctions orthogonales au système (3) lorsqu'il est incomplet.

Exemple. Pour le système $\psi_i=i\chi_1+(i-1)\chi_2+\varphi_i$, $i=1, 2, \ldots$, on a $\Delta_n(1)=\frac{n(n+1)(2n+1)}{6}$, $\Delta_n(2)=\frac{n(n-1)(2n-1)}{6}$, $\Delta_n(1, 2)=\frac{n^2(n+1)(n-1)}{12}$, donc $\frac{\Delta_n(1)+\Delta_n(2)}{\Delta_n(1, 2)}\to 0$, le système est complet.

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Manuscrit reçu
le 15.XI.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MATHÉMATIQUES

SUR LA MONOGÉNÉITÉ

Par V. S. FEDOROFF

(Présenté par N. N. Lusin, de l'Académie, le 14.XI.1944)

1. Soit α une suite finie de 2^{p-1} des nombres complexes. Nous appellerons α «le nombre du rang p» et nous allons écrire

$$\alpha = (a_1, a_2, \ldots, a_m) = [a_k, p] = (A, A'),$$

où a_K sont les nombres complexes quelconques, $m = 2^{p-1}$, A et A' sont les nombres du rang $p-1$: $A = (a_1, a_2, \ldots, a_h)$, où $h = 2^{p-2}$, et $A' = (a_{h+1}, a_{h+2}, \ldots, a_m)$. Soient α, β, γ les nombres du même rang p. Nous posons 1) $\alpha = a$, a étant un nombre complexe quelconque, à condition $a_1 = a$, $a_k = 0$ pour $k > 1$; 2) $\alpha = \beta$, où $\beta = [b_k, p]$ à condition $a_k = b_k$ pour $k = 1, 2, \ldots, m$; 3) $\alpha \pm \beta = [a_k \pm b_k, p]$; 4] $\alpha\beta = (AB + A'B', AB' + A'B)$, où $\beta = (B, B')$, et on établira de la même manière les produits AB, $A'B'$, ... de nombres A, B, A' B' du rang $p-1$, et ainsi de suite. Il est évident qu'on ait $\alpha\beta = \beta\alpha$; $(\alpha\beta)\gamma = \alpha(\beta\gamma)$; $(\alpha+\beta)\gamma = \alpha\gamma + \beta\gamma$.

Nous allons dire que α possède la propriété (Δ), s'il existe pour un nombre quelconque donné γ un tel nombre unique β, qu'on ait $\alpha\beta = \gamma$.

2. Soit D un ensemble ouvert et connexe de points $M(x_1, x_2, \ldots, x_n)$ de l'espace euclidien à n dimensions, x_i étant les coordonnées rectilignes rectangulaires de M. Nous dirons que f ou $f(M)$ est une fonction du rang p continue (harmonique,...) dans D à condition que $f = [a_k, p]$, les a_k étant des fonctions complexes de $(x_1, x_2, \ldots, x_n)$ continues (harmoniques, ...) dans D. Posons

$$\frac{\partial f}{\partial x_i} = \left[\frac{\partial a_k}{\partial x_i}, p\right]; \quad \int f\, dx_i = \left[\int a_k dx_i, p\right]$$

3. Soient f et ζ deux fonctions du même rang p continues dans D. Nous allons dire que f est monogène dans D par rapport à ζ, s'il existe dans D une fonction $f^{(1)}$ (du même rang p) de la propriété suivante; si l'on pose pour deux points quelconques M et M' de D: $r = \overline{MM'}$, $[b_k, p] = f(M') - f(M) - f^{(1)}(M) \times [\zeta(M') - \zeta(M)]$, on peut trouver pour chaque nombre $\varepsilon > 0$ (le point M étant fixe) un nombre $\delta(M, \varepsilon) > 0$ tel que, pour $r \leqslant \delta(M, \varepsilon)$, le module de chaque b_k soit inférieur à εr (on ne suppose pas *a priori* que $\delta(M, \varepsilon)$ est indépendant de la position du point M). De plus, les conditions suivantes doivent être remplies:

(a) ζ est harmonique dans D;

(b) pour chaque point de D l'une au moins des dérivées $\partial\zeta/\partial x_i$ possède la propriété (Δ);

(c) on a dans D: $\sum_{i=1}^{n} \left(\frac{\partial \xi}{\partial x_i}\right)^2 = 0.$

Cela posé, nous appellerons $f^{(1)}(M)$ la dérivée de f par rapport à ζ en M.

Remarquons qu'on peut prendre $2^{p-1} \leqslant n$ ou $2^{p-1} > n$.

4. L'auteur parvint à démontrer les propriétés suivantes de toute fonction f monogène par rapport à ζ dans D:

1) la dérivée $f^{(1)}$ est aussi monogène par rapport à ζ dans D;

2) la fonction f est déterminée d'une manière unique dans D ou bien par les valeurs de $f^{(h)}$ $(h = 0, 1 2, \ldots)$ en un point quelconque M de D, où $f^{(h+1)}$ est la dérivée par rapport à ζ de $f^{(h)}$, ou bien par les valeurs $f(M_i)$, où la suite $\{M_i\}$ converge vers M et les points M_i sont situés dans un cône, formé par des rayons issus de M et tel que pour chaque rayon L, issu de M et situé à l'intérieur ou sur la surface de ce cône, le nombre $\sum_{i=1}^{n} \alpha_i \frac{\partial \zeta}{\partial x_i}$ possède la propriété (Δ), où $\alpha_i = \cos(L, x_i)$ et $\partial\zeta/\partial x_i$ est calculé en ce point M;

3) f est harmonique dans D;

4) posons pour un nombre quelconque $\alpha = [a_k, p]$

$$|\alpha| = \sqrt[p]{|a_1|^2 + |a_2|^2 + \ldots + |a_m|^2}$$

Cela posé, si la fonction ζ est telle qu'on a en un point M de D

$$\text{(d)} \qquad |\partial^h \zeta / \partial x^h| < AR^h, \quad h = 1, 2, 3, \ldots$$

(où x désigne une telle coordonnée x_i pour laquelle $\partial\zeta/\partial x_i$ possède la propriété (Δ) et les constantes A et R sont indépendantes de h), la fonction f est analytique par rapport à ζ en ce point M, c'est-à-dire on a dans un certain voisinage de M

$$f = \sum_{h=0}^{\infty} \frac{1}{h!} f_0^{(h)} (\zeta - \zeta_0)^h \qquad (1)$$

où $f_0^{(h)} = f^{(h)}(M)$, $h = 0, 1, 2, 3 \ldots$, $\zeta_0 = \zeta(M)$ et la série (1) est absolument convergente dans ce voisinage.

Notons enfin que certains théorèmes de G. Scheffers [1] et P. W. Ketchum [2] sont analogues aux théorèmes de cette Note (cf. aussi J.A. Ward [3], où on trouve une bibliographie complète).

Manuscrit reçu
le 14. XI. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MATHEMATICAL PHYSICS

ON THE THEORY OF THE CHARACTERISTIC FREQUENCIES OF POLYATOMIC MOLECULES

By L. S. MAJANTZ

(Communicated by V. A. Fock, Member of the Academy, 9.VIII.1944)

It was shown by the author in previous works [1,2] that if a molecule is considered as a system of N material points under the action of internal forces, the frequencies and the modes of its normal vibrations may be obtained by solving the equation

$$W\vec{r} = \lambda\vec{r} \tag{1}$$

where $W = K^{-1}V$, K and V are the linear Hermitian operators defined by the equations

$$(\vec{q}, Kq) = \sum_{i,j=1}^{3N} t_{ij}\dot{q}^i\dot{q}^j, \quad (\vec{q}, V\vec{q}) = \sum_{i,j=1}^{3N} u_{ij}q^iq^j$$

$\vec{r}$ and λ are defined by the conditions $\vec{q} = \vec{r}\cos(\sqrt{\lambda}\,t + \varphi)$ (*i. e.* λ is equal to the square of the angular frequency of the vibration, and $\vec{r} = \sum_{i=1}^{3N} r^i\vec{e}_i$ is the vector which determines the mode of the vibration); $\vec{q} = \sum_{i=1}^{3N} q^i\vec{e}_i$ is the vector which determines the spatial state of the system; q^i $(i = 1, 2, \ldots, 3N)$ are the generalized coordinates; $\vec{e}_i$ $(i = 1, 2, \ldots, 3N)$ is the space basis; $\frac{1}{2}\sum_{i,j=1}^{3N} t_{ij}\dot{q}^i\dot{q}^j$ is the kinetic energy of the system; $\frac{1}{2}\sum_{i,j=1}^{3N} u_{ij}q^iq^j$ is the potential energy of the system.

The matrices of the operators have the following form (using the same notation as for the corresponding operators):

$$K = G^{-1}T; \quad V = G^{-1}U \quad \text{and} \quad W = T^{-1}U$$

where $T = \|t_{ij}\|$; $U = \|u_{ij}\|$ and $G = \|g_{ij}\|$ with $g_{ij} = (\vec{e}_i, \vec{e}_j)$.

The values of λ differing from zero are obtained on solving equation (1) for the operator derived from the operator W in the subspace corresponding to the internal coordinates, whose number is $n=3N-6$ (or $3N-5$, if the equilibrium configuration is linear). The matrix of this operator is equal to the product of the submatrices T^{-1} and U corresponding to this subspace.

Consequently, if $T^{-1}=\|\tau_{ij}\|$ and $W=\|w_{ij}\|$, then $w_{ij}=\sum_{k=1}^{n}\tau_{ik}u_{kj}$.

In what follows we shall assume that equation (1) refers to internal coordinates.

Let $\vec{r}_{i'}$ be the proper vectors of the operator W and $\lambda_{i'}$ $(i'=1,2,\ldots n)$, their corresponding eigenvalues. We will consider a subspace of n' dimensions corresponding to the internal coordinates of some group of k atoms $(2\leqslant k\leqslant N)$. Let the frequencies and modes of the normal vibrations of this isolated group (assuming that the parameters of the group retain the values they had in the molecule) be determined by the proper vectors $\vec{p}_i$ and the eigenvalues ν_i $(i=1,\ 2,\ldots,n')$ of the corresponding operator $W_0=K_0^{-1}V_0$ which acts in this subspace. If for some values of l' and l $\lambda_{l'}$ does not differ much from ν_l, one may speak of a characteristicity of the frequency of the l-th normal vibration of the whole system for the given group. To evaluate this characteristicity quantitatively we have introduced ([3]) the quantity

$$\alpha_l=\frac{\lambda_{l'}-\nu_l}{\nu_l}$$

which we have called the coefficient of the incharacteristicity vibration frequency. If for some values of l' and l the direction of the vector $\vec{r}_{l'}$ does not differ much from that of the vector $\vec{p}^{l}$, one may speak of a characteristicity of the mode of the l'-th normal vibration of the whole system for the given group. To evaluate this characteristicity quantititavely one may introduce the quantity

$$\beta_l=\operatorname{tg}\left(\vec{r}_{l'}\vec{p}_l\right)=\frac{\sqrt{(\vec{r}_{l'}\vec{r}_{l'})\cdot(\vec{p}_l,\ \vec{p}_l)-(\vec{r}_{l'},\ \vec{p}_l)^2}}{(\vec{r}_{l'},\ \vec{p}_l)}$$

(with a certain choice of G) and call it the coefficient of incharacteristicity of the vibration mode.

It should be noted that the characteristicity of the mode and the frequency of the vibration that refers to each individual molecule should not be confused with the conservation of the characteristicity on passing from one molecule to another having the same group of atoms. Historically it was precisely this conservation of the characteristicity for certain groups of atoms that was discovered experimentally and led to the introduction of the conception of characteristic frequencies.

Let us now consider the application of the perturbation medhod ([4]) to the solution of equation (1) in its relation to problems of characteristicity.

Let us select as a part of the internal coordinates the changes in the lengths of the bonds between the atoms and the independent variations of definite valence angles multiplied by the geometric mean of the lengths of their sides. Let us further assume that with this choice of coordinates $G=E$.

To consider the characteristicity with respect to an individual chemical bond ($k=2$) we shall take as an unperturbed operator, W_0, the operator whose matrix has the following form in these coordinates:

$$W_0 = \| \tau_{li} u_{ii} \delta_{ij} \|$$

Then the original basic vectors $\vec{e_i}$ are the proper vectors of W_0 with the proper values $\nu_i = \tau_i u_{ii}$ (*i. e.* ν_i is equal to the square of the angular frequency of the vibration of the i-th isolated coordinate). Here $w'_{ij} = w_{ij} = \sum_k \tau_{ik} u_{kj}$ and $w'_{ii} = \sum_{k \neq i} \tau_{ik} u_{ki}$.

Since $G = E$ by assumption, the coefficient of incharacteristicity of the vibration mode assumes the following form

$$\beta_l = \frac{\sqrt{\sum_{i \neq l}^{i} (r_{il}')^2}}{r'_{ll}}$$

On the basis of equation (3b') [4] we conclude that the necessary condition for $|\beta_l| \leqslant b$ is

$$\frac{|w'_{il}|}{\sqrt{(\lambda_{l'} - \nu'_i)^2 + \sum_{j \neq i,l} (w'_{ij})^2}} \leqslant b \quad (i \neq l) \tag{I'}$$

where $\nu'_i = \nu_i + w'_{ii}$; or if

$$|\Delta \nu'_l| \ll |\nu'_l - \nu'_i| \tag{a}$$

$$\frac{|w'_{il}|}{\sqrt{(\nu'_l - \nu'_i)^2 + \sum_{j \neq i,l} (w'_{ij})^2}} \leqslant b(1 \pm \varepsilon) \quad (i \neq l) \tag{I''}$$

where ε is small in comparison with unity.

A condition sufficient for this when it is possible to employ the «perturbation method» [4] is

$$\sqrt{\sum_{j \neq l} \left(\left| \frac{w'_{jl}}{\lambda_{l'} - \nu'_j} \right| + \rho'_{(\infty,l,j)} \right)^2} \leqslant b \tag{II'}$$

or, when (a) is fulfilled

$$\sqrt{\sum_{j \neq l} \left(\left| \frac{w'_{jl}}{\nu'_l - \nu'_j} \right| + \rho'_{(1,l,j)} \right)^2} \leqslant b(1 \pm \varepsilon) \tag{II''}$$

In order that the coefficient of incharacteristicity of the vibration frequency (ν_l) be small when the coefficient of incharacteristicity of the vibration mode (β_l) is small, it is necessary that (w'_{ll} be sufficiently small as compared with ν_l. This condition may be fulfilled owing to the specific character of the chemical forces, namely, the fact that the elements of the

matrix U may be subdivided according to magnitude into a number of groups, the diagonal elements corresponding to the bonds being the largest; the diagonal elements corresponding to the valence angles will be smaller; the non-diagonal elements corresponding to the connected coordinates will be still smaller, while the non-diagonal elements corresponding to the unconnected coordinates can in most cases evidently be regarded as equal to zero. This specificity may lead to the condition that $w'_{ll} = \sum_{k \neq l} \tau_{lk} u_{kl}$ will indeed be small in comparison with $\nu_l = \tau_{ll} u_{ll}$ (for l referring to the bond).

The characteristicity with respect to a group of atoms ($k > 2$) may be considered in an analogous way. In this case one may select as an unperturbed operator W_0, the operator that leaves invariant the subspace of n' dimensions corresponding to this group of atoms and has in this subspace a matrix equal to the product of the submatrices T^{-1} and U corresponding to this subspace, while in the rest of the space it has a matrix with elements equal to $\tau_{ii} u_{ii} \delta_{ij}$. The expression for β_l in this case becomes somewhat more complex, since the proper vectors of W_0 lying in the subspace examined are, generally speaking, no longer orthogonal to each other.

Received
9. VIII.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII. № 6

OCEANOLOGY

DIFFERENCES BETWEEN ACCUMULATIVE AND ABRASIONAL SHORELINE PROFILES

By V. P. ZENKOVICH

(Communicated by P. P. Shirshov, Member of the Academy, 3.VIII.1944)

An opinion which has taken firm root in literature, is that the shape of the equilibrium profile curve of sea-shores does not depend upon the composition of the bottom. According to this view, in the maturity stage the profiles of both accumulative and abrasional sections are shaped concavely upwards and the curvature of the profile in both cases is maximum at the shore, decreasing with the depth ((1) pp. 203 and 217). An analysis of the processes which take place on the underwater shore slope, however, shows that this view is incorrect.

If the underwater slope and the shore are made up of debris with the particle size sufficiently large to prevent breakers from lifting it into suspension, then, as we know, transverse shifting of debris with relation to the shore will cease as soon as the equilibrium profile has been reached (2). At this stage under the action of the disturbance the debris particles at each point of the bottom continue to oscillate with an amplitude which, for equal size of the debris particles, increases abruptly toward the shore. If the wave spreads along a normal to the shore, then both the directions and the segment lengths of these oscillations are identical, with the result that the debris particles do not move from their positions on the bottom. If the wave approaches the shore at an acute angle, each direction has its own equilibrium profile, and after this profile has been reached, shifting of the debris will take place only in a longitudinal and not in a transverse direction.

At this stage the bottom slopes the most steeply at the shore, since it is here that the inequality of the wave motion velocities of the water is greatest, being compensated on the steep part of the slope by the action of the gravity of the debris particles. This inequality in the amplitudes of the motions of the debris grains shows that under such conditions as are discussed here the wave energy expenditure also increases abruptly toward the shore. On the whole, this parabolical curve, representing the profile of the bottom, is a singular kind of wave energy damper.

Now suppose that the bottom and the shore are formed of primary rock, which the waves partly erode into a comparatively small quantity of rock fragments, not enough to form a continuous layer over the whole of the bottom. Then we may conclude that the velocity of erosion of the rock will be in proportion to the quantity of energy developed by the wave at the given point of the bottom.

The same relationship should be true for the development of a profile made up of small-grained debris, which the action of the waves brings into suspension, and which does not settle down on the bottom even during the pauses between wave motions. The suspension formed in this case by the counter-current at the bottom is carried down the underwater slope to a greater depth. Evidently, the quantity

of material in suspension is also in proportion with the wave energy, and the profile formed in this case should be analogous in form to the abrasional profile in primary rock.

As to the shape of this profile, both field observation and experimental data show that it should be a very gently sloping curve, bent convexly and not concavely upwards, *i. e.* its shape should be exactly the opposite to the curve mentioned above for the profile developed in large-grained debris.

Sand embankments on the open sea give a real curve of the first kind. In this case, as a rule, the slope going out from the shore is a very gentle one (less than 1 : 100), which eventually becomes steeper with the depth, and then becomes gentler again at the base of the underwater shore slope. Abrasional shores in primary rocks with small quantities of debris also form very gently sloping bench platforms everywhere, instead of the concave curve with its slope increasing toward the shore, postulated by Johnson (1).

In tank experiments when the sand embankment is made with an even slope, the action of the waves, independent of the steepness of the slope, is always greatest in some definite narrow zone, in which the bottom is eroded (2).

That the equal energy curve should be bent convexly upward, can be concluded from an examination of the equation of wave motion at small depths. But it is impossible, however, to deduce mathematically the true shape of the curve, since the problem of the energy loss of a wave due to friction against the bottom under various conditions of slope and depth (in relation to the parameters of the wave), and for various compositions of the bottom, has not been investigated thoroughly enough as yet. So far, only empiric relationships have been worked out for this problem (4).

Thus, when we speak about equilibrium profiles, we should picture two qualitatively different shapes. The first is the curve of a profile for debris of a g i v e n p a r t i c l e s i z e; the second is the e q u a l e n e r g y e x p e n d i t u r e curve. The parameters of the latter curve depend upon that minimum quantity of energy which can still bring about erosion of the rock or lift the debris into suspension. In concluding, it should be noted that owing to constant grinding of the debris as it is moved from place to place, the first profile in time is gradually rearranged and in the limit (at the stage of maturity or old age) it should change into the second type.

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Received
3.VIII.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

TECHNICAL PHYSICS

PROPAGATION OF SOUND WAVES IN A LIQUID LAYER BETWEEN TWO ABSORBING HALF-SPACES

By L. M. BREKHOVSKIKH

(Communicated by S.I.Vavilov, Member of the Academy, 31.VIII.1944)

This problem may find application in geophysics (*e. g.* propagation of sound through an oil layer), in architectural acoustics (propagation of sound between floor and ceiling in buildings whose walls are far apart or highly absorbent) and in hydroacoustics (propagation of sound through the sea). In the latter case one of the half-spaces is air (or, for all practical purposes, a void). This case was investigated by D. I. Sherman under the assumption that the wave source is at the surface of the liquid or at the bottom and that in an elastic half-space waves are propagated without damping. In our theory we assume that the source is located at any point and in contrast to Scherman maintain that damping in the half-space is great in comparison with damping in the liquid. We have no reliable experimental data for the absorption of sound in soils, which refute or support this hypothesis. However, some preliminary experiments carried out by the Acoustics Laboratory at the Physical Institute of the Academy of Sciences of the USSR seem to indicate that this absorption is great, at least within a frequency range of 50—1 000 cycles*.

We shall consider here only the sinusoidal regime of the source. The cylindrical system of coordinates is so selected that the plane $z=0$ coincides with the boundary of the lower half-space. Let h be the thickness of the layer; $(0, z_0)$, the coordinates of the source, and (r, z), the coordinates of the receiver. The index 0 denotes the liquid, and the indices 1 and 2, the lower and upper half-spaces. Let the media be characterized by the density ρ_j and the Lamé elasticity constants λ_j, μ_j. When $j=1, 2$, the latter are assumed to be complex owing to absorption. For liquids, obviously, $\mu_0=0$, while λ_0 will be regarded as real. The potentials of sound waves φ_j $(j=0, 1, 2)$ and of transverse waves ψ_j $(j=1, 2)$ are given by the usual equations ((2), ch. XII, § 5, 7)

$$\frac{\partial^2\varphi_j}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_j}{\partial r}+\frac{\partial^2\varphi_j}{\partial z^2}+k_j^2\varphi_j=0 \qquad (1)$$

$$\frac{\partial^2\psi_j}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_j}{\partial r}+\frac{1}{r^2}\psi_j+\frac{\partial^2\psi_j}{\partial z^2}+\varkappa_j^2\psi_j=0 \qquad (2)$$

* In exactly the same way as described below the author has solved the problem of the propagation of radio waves between two absorbing half-spaces.

where

$$k_j^2 = \frac{\omega^2}{\lambda_j + 2\mu_j}\rho_j, \qquad \varkappa_j^2 = \frac{\omega^2 \rho_j}{\mu_j} \tag{3}$$

Here the velocity components are

$$v_{jr} = -\frac{\partial \varphi_j}{\partial r} + \frac{\partial \psi_j}{\partial z}; \quad v_{jz} = -\left(\frac{\partial \varphi_j}{\partial z} + \frac{\partial \psi_j}{\partial r} + \frac{\psi_j}{r}\right) \tag{4}$$

and the sound pressure in the liquid $p_0 = i\omega\rho_0\varphi_0$ (the relation between all the quantities and time is given by the factor $e^{i\omega t}$ which is omitted henceforth).

Our source will be a pulsating sphere of small radius with a given volume velocity V_0. Henceforth it will be assumed for simplicity that $V_0/4\pi = 1$. At points close to the source, where the influence of the boundaries may be discarded, we should have

$$\varphi_0 = \frac{e^{ik_0 R}}{R} \tag{5}$$

where $R = \sqrt{(z - z_0)^2 + r^2}$. When R is small, the solution of our equations should reduce to (5). Furthermore, at each of the two boundaries the solutions should satisfy the conditions of continuity: a) of the normal velocities v_z, b) of the normal component of this tensor σ_z, c) of the tangential component of this tensor τ_{zr}. Expressing σ_z and τ_{zr} through the potentials ([2], Ch. VII, §§ 2, 3), we get the conditions at the boundary $z = 0$

$$\left.\begin{aligned} &\frac{\partial \varphi_0}{\partial z} = \frac{\partial \varphi_1}{\partial z} + \frac{\partial \psi_1}{\partial r} + \frac{1}{r}\psi_1; \quad 2\frac{\partial^2 \varphi_1}{\partial r\, \partial z} - 2\frac{\partial^2 \psi_1}{\partial z^2} - \varkappa_1^2 \psi_1 = 0 \\ &\lambda_0 k_0^2 \varphi_0 = \lambda_1 k_1^2 \varphi_1 - 2\mu_1 \left[\frac{\partial^2 \varphi_1}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{\partial \psi_1}{\partial r} + \frac{1}{r}\psi_1\right)\right] \end{aligned}\right\} \tag{6}$$

The conditions for $z = h$ can be found from this by replacing the index 1 by 2 and $\partial/\partial z$ by $-\partial/\partial z$.

Owing to the cylindrical symmetry of the problem, it is natural to seek the solution of our equations in the form of a development into cylindrical functions. Hence we get:

$$\left.\begin{aligned} z < z_0, \varphi_0 &= \int_0^\infty M_1(\xi, z, z_0) J_0(\xi r)\, d\xi \\ z > z_0, \varphi_0 &= \int_0^\infty M_2(\xi, z, z_0) J_0(\xi r)\, d\xi \end{aligned}\right\} \tag{7}$$

$$\varphi_1 = \int_0^\infty \Phi_1(\xi) J_0(\xi r) e^{b_1 z} d\xi; \quad \psi_1 = \int_0^\infty \Psi_1(\xi) e^{d_1 z} J_1(\xi r)\, d\xi \tag{8}$$

and the same for φ_2 and ψ_2 with b_1 and d_1 replaced by $-b_2$ and $-d_2$ and Φ_1 and Ψ_1 by Φ_2 and Ψ_2. Here

$$b_j = \sqrt{\xi^2 - k_j^2}, \quad d_j = \sqrt{\xi^2 - \varkappa_j^2}, \quad j = 0, 1, 2 \tag{9}$$

We assume the signs before the roots to be such that the real parts of b_j and d_j be positive.

It can easily be verified that the solutions of (7) and (8) satisfy the equations, the condition at the source, and the boundary conditions, and disappear at infinity if the functions $M_1, M_2, \Phi_1, \ldots$ are chosen thus

$$M_1 = \frac{2\xi}{\Delta b_0}\left[\operatorname{sh} b_0 z + A_1 g_1 \operatorname{ch} b_0 z\right]\left[\operatorname{sh} b_0 (h - z_0) + A_2 g_2 \operatorname{ch} (h - z_0)\right] \tag{10}$$

M_2 can be obtained from M_1 by replacing z by z_0 and *vice versa*. Here

$$A_j = \frac{1}{\varkappa_j^4}\left[(2\xi^2 - \varkappa_j^2)^2 - 4\varkappa_j b_j d_j \xi\right]; \quad g_j = \frac{b_0 \rho_j}{b_j \rho_0} \tag{11}$$

$$N = A_1 g_1 \operatorname{ch} b_0 h + \operatorname{sh} b_0 h + A_2 g_2 (A_1 g_1 \operatorname{sh} b_0 h + \operatorname{ch} b_0 h) \tag{12}$$

Further

$$\Phi_1 = \frac{2\xi(\varkappa_1^2 - 2\xi^2)}{b_1 \varkappa_1^2 \Delta}\left[A_2 g_2 \operatorname{ch} b_0 (h - z_0) + \operatorname{sh} b_0 (h - z_0)\right] \tag{13}$$

$$\Psi_1 = \frac{4\xi^3}{\varkappa_1^2 \Delta}\left[A_2 g_2 \operatorname{ch} z_0 (h - z_0) + \operatorname{sh} b_0 (h - z_0)\right] \tag{14}$$

Φ_2 and Ψ_2 can be determined by replacing the index 1 by 2 and $h - z_0$ by z_0.

Substituting (10) and (13) into (7) and (8) we arrive at a solution in integral form. It is convenient to replace the integral from 0 to ∞ from the Bessel function in it by half an integral from $-\infty$ to $+\infty$ from the Hankel function of the same order of the first kind.

Let us extend the path of integration from the real axis to the upper half of the complex plane*. Then each of the integrals in the solution will be equal to the sum of remainders in the poles in the upper half-plane plus the integrals along the edges of the section (the terms under the integral signs are multi-valued functions with their branching points $\xi = \pm k_j, \pm \varkappa_j$, $j = 1, 2$). The integrals along the sections are found to give waves which spread with the same damping as the waves in the half-planes. We assume this damping to be large in comparison with that of the waves spreading through the layer and hence this part of the integrals may be neglected. It is possible to do this only in the case where a field is considered in a layer or in a half-space at a distance from the layer which is small in comparison with the distance to the source. This condition is also assumed to be fulfilled.

As a result we get for the potential in the layer

$$\varphi_0 = \pi i \sum_{n=1}^{\infty} \operatorname{Res}(M_1)_{\xi=\xi_n} H_0^{(1)}(\xi_n r) \equiv \pi i \sum_{n=1}^{\infty} \operatorname{Res}(M_2)_{\xi=\xi_n} H_0^{(1)}(\xi_n r) \tag{15}$$

where ξ_n $(n = 1, 2, \ldots)$ represents all those roots of the transcendental equation $N = 0$ which have a positive imaginary part.

Solutions in the semi-spaces give the same expressions, except that the value for φ_1 will be Res $(\Phi_1) e^{b_1 n z}$ instead of Res (M_1) and for ψ_1 Res $(\Psi_1) e^{d_1 n z}$, and analogously for φ_2 and ψ_2 with index 1 replaced by 2 and a minus sign in the exponents. Here $b_{j_n} = \sqrt{\xi_n^2 - k_j^2}$ and analogously for d_{j_n}.

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* Here the expressions under the integral signs have an infinite number of poles and among them infinity, to which we extend the path of integration. In order to be able to apply the Cauchy theorem in this case, it is necessary that a number of conditions imposed on the function itself at infinity and on the position of its poles be fulfilled. An investigation not given here showed that these conditions were fulfilled in our case.

Thus the solutions in their final form are series in Hanckel functions. They are found to converge well if the thickness of the layer is not very great, if, say, $k_0 h$ is not over 10 or 20. The main difficulty in the problem, evidently, reduces to finding the roots ξ_n of the equation $N=0$ (see (12)).

In the case of propagation of sound in the sea, the upper half-space may be regarded as a void. Then $\rho_2=0$ and taking into account (10) we get from (15)

$$\varphi_0 = -2\pi i \sum_n \left[\frac{\operatorname{sh} b_0 (h-z_0) \operatorname{sh} b_0 (h-z)}{h + \operatorname{ch}\ b_0 h \cdot \frac{\rho_1 S}{\rho_0 \varkappa_1^4}} \right]_{\xi=\xi_n} H_0^{(1)} (\xi_n r) \tag{16}$$

where

$$S = \frac{(2\xi^2 - \varkappa_1^2)}{b_1^3} [(2\xi^2 - \varkappa_1^2)(k_0^2 - k_1^2) + 8 b_0^2 b_1^2] - \frac{4}{d_1} (\xi^2 d_1 + \xi^2 b_0 + 2 b_0^2 d_1^2) \tag{17}$$

and ξ_n are the roots of the equation

$$A_1 g_1 \operatorname{ch} b_0 h + \operatorname{sh} b_0 h = 0 \tag{18}$$

The main difficulty in this problem, evidently, consists in solving equation (18) in the complex field. This solution can be arrived at comparatively easily in some particular cases, *e. g.* when h is small, or, on the contrary, when h is large. However, these solutions are based on certain assumptions as to the elastic characteristics of soils, which cannot as yet be regarded as experimentally grounded and hence are not given here.

These formulae can be simplified considerably by simplifying the boundary conditions, which can be done by employing the so-called normal impedance [1]. This simplification consists in neglecting ξ^2 in all cases in comparison with k_j^2 and $\varkappa_j^2$, which is possible if the velocity of propagation of the elastic waves in the half-space is much lower than in the liquid, or if the absorption in them is great already at the length of the wave. In thisit case it appears that waves are no longer excited in the half-space. Meanwhile, however, we do not as yet know whether these requirements are satisfied or not in the case of sea grounds.

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Received
31.VIII.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

ELECTRICITY

SUBSTITUTION DIAGRAMS FOR TRANSIENT REGIMES OF ASYNCHRONOUS ELECTRIC MACHINES

By P. A. SVIRIDENKO

(Communicated by K. I. Shenfer, Member of the Academy, 25.IX.1944)

Substitution diagrams greatly simplify research work concerned with rotary electric mechanisms. During the last few decades, the well-known equivalent diagram for stationary regimes of asynchronous machines has been extensively used. This diagram is in principle inapplicable to transient processes in asynchronous machines (sudden short circuits, switching-on of voltage, sudden load changes, *etc.*).

The present paper shows that for transient regimes analysis of an asynchronous rotary electric machine can be substituted by analysis of a statical electric diagram.

To solve this problem we proceed as follows: we write the differential equations of a real rotary electric machine under certain assumptions and then select such a statical electric circuit whose differential equations are identical with those written. The coordinate system selected for plotting the real currents and voltages in the windings of the machine, must make it possible to keep the parameters of the equivalent diagram unchanged (for linear problems).

For this purpose let us take a component system α, β, 0, which will permit us to write the differential equations for the machines with constant and not periodical coefficients ([1], [2]).

Stanley's initial system of equations is

$$\left.\begin{aligned}
&L_0\frac{dI_\alpha}{dt}+M_0\frac{di_\alpha}{dt}+RI_\alpha+v_\alpha\\
&L_0\frac{dI_\beta}{dt}+M_0\frac{di_\beta}{dt}+RI_\beta=v_\beta\\
&M_0\frac{dI_\alpha}{dt}+M_0\omega_p I_\beta+l_0\frac{di_\alpha}{dt}+l_0\omega_p i_\beta+ri_\alpha=0\\
&-M_0\omega_p I_\alpha+M_0\frac{di_\beta}{dt}+l_0\frac{di_\beta}{dt}-l_0\omega_p i_\alpha+ri_\beta=0
\end{aligned}\right\}\qquad(1)$$

Here I_α and I_β, are the stator currents on the axes α and β; i_α and i_β, the rotor currents on the axes α and β; v_α and v_β, the voltage applied to the stator on the axes α and β; L_0, l_0 and M_0, the seeming coefficients of inductance and mutual induction in the 3-phase stator and rotor; R and r, the active

resistance of the stator and rotor; ω_p, velocity of the rotor in electrical radians per second.

First, let us consider an asynchronous machine with one phase on the stator and two on the rotor (we assume the rotor to be electrically balanced). If we now apply the system (1) or equations for a 3-phase machine to our case, assuming the velocity of the rotor to be constant under transient regimes, and solve the system of equations with respect to the currents we get

$$\left.\begin{aligned} A\frac{d^3I_\alpha}{dt^3}+B\frac{d^2I_\alpha}{dt^2}+C\frac{dI_\alpha}{dt}+DI_\alpha &= (\omega_p^2l^2+r^2)\,v_\alpha+2rl\frac{dv_\alpha}{dt}+l^2\frac{d^2v_\alpha}{dt^2} \\ A\frac{d^3i_\alpha}{dt^3}+B\frac{d^2i_\alpha}{dt^2}+C\frac{di_\alpha}{dt}+Di_\alpha &= -Ml\omega_p^2v_\alpha-Mr\frac{dv_\alpha}{dt}-Ml\frac{d^2v_\alpha}{dt^2}, \\ A\frac{d^3i_\beta}{dt^3}+B\frac{d^2i_\beta}{dt^2}+C\frac{di_\beta}{dt}+Di_\beta &= Mr\omega_pv_\alpha \end{aligned}\right\} \qquad (2)$$

where

$$A=(Ll-M^2)\,l, \qquad B=[Rl^2+r\,(2Ll-M^2)],$$
$$C=[r\,(2Rl+Lr)+l\omega_p^2(Ll-M^2)], \qquad D=R\,(\omega_p^2l^2+r^2)$$

Equations of the third order (2) are linear and have constant coefficients.

Now let us select a statical electric diagram with constant parameters whose differential equations have a structure analogous to (2).

For each current component I_α, i_α and i_β such a substitutional diagram, which takes into consideration not only all the inductive but also the active resistance of the stator and rotor (the presence of terms with the third, second, first and zero derivatives of the current) can be represented, for instance, as two statical, inductively connected circuits or as the corresponding diagrams with two electrically connected contours. The first circuit of this diagram, to which external voltage is applied, corresponding to the right-hand side of equation (2), consists of active and inductive resistance and the second (a closed circuit) besides inductive and active resistance has also capacitance*.

The values of the parameters of such a diagram can be determined from the equality of the coefficients of derivatives of the same order in the equations of the machine and those of the diagram, which must satisfy the condition of equality. Thus, the machine under consideration with a one-phase stator and a two-phase rotor can be substituted by three identical substitution diagrams for currents I_α, i_α and i_β.

Now if the active resistance in the stator windings is neglected, the last terms in the left-hand sides of equations (2) disappear. Integrating equations (2) term by term from zero to «t» and assuming that $v_\alpha=v_m\cos\omega t$ we get

$$A_1\frac{d^2I_\alpha}{dt^2}+B_1\frac{dI_\alpha}{dt}+C_1I_\alpha=v_m\frac{k}{\omega}\cos(\omega t+\varphi) \qquad (3)$$

where

$$A_1=(Ll-M^2)\,l; \qquad B_1=r\,(2Ll-M^2),$$
$$C_1=r^2L+l\omega_p^2(Ll-M^2),$$
$$k=\sqrt{[l^2(\omega^2-\omega_p^2)-r^2]^2+4r^2l^2\omega^2}$$
$$\operatorname{tg}\varphi=\frac{l^2(\omega^2-\omega_p)-r^2}{2rl\omega}$$

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* Without this capacitance the number of contours necessary would be increased to three.

In this manner, we reduce the order of the differential equation by one, and it becomes possible to compose a simpler equivalent single contour diagram consisting of active resistance, capacitance and inductance (in the double-contour diagram the capacitance is excluded).

In this diagram for the current I_α, the equivalent parameters equal

$$\left.\begin{aligned} R_э &= \frac{B_1\omega^2}{k} = \frac{r(2Ll - M^2)\omega^2}{k}, \\ L_э &= \frac{A_1\omega^2}{k} = \frac{(Ll - M^2)l\omega^2}{k}, \\ \frac{1}{C_э} &= \frac{C_1\omega^2}{k} = \frac{r^2L\omega^2}{k} + \frac{\omega_p^2 l}{k}(Ll - M^2)\omega^2 \end{aligned}\right\} \qquad (4)$$

and the applied voltage

$$\varphi = \varphi_m \sin(\omega t + \varphi)$$

The above parameters of the substitution diagrams provide the possibility of easily determining the nature of the course of free and compulsory processes in an asynchronous machine.

Passing over to a three-phase asynchronous machine we find that the structure of the equations and the form of the substitution diagrams remain unchanged. Assuming for simplicity to begin with that $R = r = 0$ and solving system (1) for each current component we get:

$$\left.\begin{aligned} \frac{L_0l_0 - M_0^2}{l_0}\frac{d^3I_\alpha}{dt^3} + \frac{(L_0l_0 - M_0^2)\omega_p^2}{l_0}\frac{dI_\alpha}{dt} &= \omega_p^2\varphi_\alpha + \frac{d^2v_\alpha}{dt^2} \\ \frac{L_0l_0 - M_0^2}{l_0}\frac{d^3I_\beta}{dt^3} + \frac{(L_0l_0 - M_0^2)\omega_p^2}{l_0}\frac{dI_\beta}{dt} &= \omega_p^2\varphi_\beta + \frac{d^2v_\beta}{dt^2} \end{aligned}\right\} \qquad (5)$$

$$\left.\begin{aligned} \frac{L_0l_0 - M_0^2}{M_0}\frac{d^3i_\alpha}{dt^3} + \frac{(L_0l_0 - M_0^2)\omega_p^2}{M_0}\frac{di_\alpha}{dt} &= -\omega_p^2\varphi_\alpha - \frac{d^2v_\alpha}{dt^2} \\ \frac{L_0l_0 - M_0^2}{M_0}\frac{d^3i_\beta}{dt^3} + \frac{(L_0l_0 - M_0^2)\omega_p^2}{M_0}\frac{di_\beta}{dt} &= -\omega_p^2\varphi_\beta - \frac{d^2v_\beta}{dt^2} \end{aligned}\right\}$$

Reducing the order of these equations by one we find that for each of the four current components of the stator and rotor of the 3-phase asynchronous machine we can draw up an equivalent diagram consisting only of inductance and capacitance ($R_э = 0$), since we assumed $R = r = 0$).

The values of the equivalent parameters and the applied voltage equal:

1. In the diagram for current I_α

$$L_э = \frac{(L_0l_0 - M_0^2)\omega^2}{l_0(\omega^2 - \omega_p^2)}; \qquad \frac{1}{C_э} = \frac{(L_0l_0 - M_0^2)\omega_p^2\omega^2}{l_0(\omega^2 - \omega_p^2)}$$

$$\varphi = \varphi_\alpha = +\varphi_m \cos\omega t^*$$

2. In the diagram for current I_β, $L_э$ and $C_э$ have the same values as in case one, and

$$\varphi = \varphi_\beta = -\varphi_m \sin\omega t$$

In the diagrams for currents i_α and i_β, $L_э$, $C_э$ and φ are found in an analogous manner.

These correlations and this substitution diagram which consists only of capacitance and inductance can represent only an «ideal» three-phase machine with super-conductive rotor (and stator) windings. Such a machine has no slip (becomes synchronous).

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* For symmetrical voltages applied to the stator.

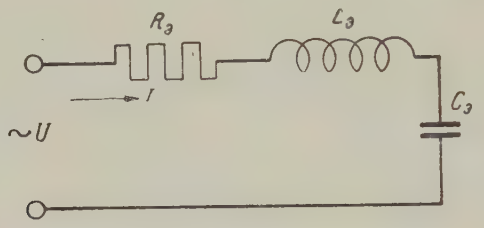
A real 3-qhase asynchronous machine with active resistance in its windings gives differential equations of the fourth order. In this case the substitution diagram for each current (I_α, I_β, i_α and i_β) can be represented, for instance, as two connected contours, each of which contains R, L and C. However, if we neglect only the active resistance of the stator (which is quite permissible), then from this equation we can eliminate the terms with the first and second derivatives of the current and so reduce the equation to the second order

$$A\frac{d^4I_\alpha}{dt^4}+B\frac{d^3I_\alpha}{dt^3}+C\frac{d^2I_\alpha}{dt^2}=a\frac{dv_\alpha}{dt}+b\frac{d^2v_\alpha}{dt^2}+c\frac{d^3v_\alpha}{dt^3}+d\frac{dv_\beta}{dt} \qquad (6)$$

where

$$A=(L_0l_0-M_0^2)^2, \qquad B=2L_0r(L_0l_0-M_0^2), \qquad C=L_0^2r^2+(L_0l_0-M_0^2)^2\omega_p^2,$$
$$a=L_0r^2+(L_0l_0-M_0^2)\,l_0\,\omega_p^2, \qquad b=r(2L_0l_0-M_0^2),$$
$$c=(L_0l_0-M_0^2)\,l; \quad d=M_0^2r\omega_p$$

Equation (6) shows that for a three-phase asynchronous machine the equivalent diagrams for the currents can be represented in the same form as above (see figure). The values of the equivalent parameters of the



Substitution diagram for transient processes in asynchronous machines

diagrams and the applied voltages can be found as before, from the coefficients of the equations and the given voltage on the terminals of the machine.

These results give grounds to conclude that a rotary asynchronous machine in its transient processes can be represented by statical electrical diagrams and that each equivalent parameter of the diagram has not one, but several corresponding real parameters in the machine, combined in some definite manner.

In conclusion it should be noted that equivalent diagrams in this sense can be obtained in an analogous manner for asynchronous machines with any impedance in series with the stator circuit by using the generalized equations (2) and also for 3-phase alternating current machines with slip-rings on the basis of the fundamental equations proposed by the author ([3]).

Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

CHEMISTRY

CHEMICAL CHANGES WHICH TAKE PLACE IN THE CELLULOSE MACROMOLECULE IN COPPER-AMMONIA SOLUTION UNDER THE ACTION OF ATMOSPHERIC OXYGEN

By V. I. IVANOV and E. D. KAVERZNEVA

(Communicated by A. N. Nesmeyanov, Member of the Academy, 16.VI.1944)

A study of the change undergone by the cellulose macromolecule when acted upon with oxidizing agents is of great scientific interest, since even mild oxidizers sometimes cause considerable changes in its chemical, physical-chemical and physical-mechanical properties. The mechanism of the decomposition of cellulose when oxidized has not as yet been adequately elucidated. It has only been pointed out that the starting point of the oxidation process may be, depending on the nature of the oxidizing agent and the conditions of the reaction, the sixth carbon atom in the glucose radicals and the bond between the second and third carbons in the pyrane ring. The causes for the rapid depolymerization of the oxidized cellulose in alkaline solution have not as yet been found.

We have studied the oxidation of cellulose with atmospheric oxygen in a copper-ammonia solution, which is almost the only solvent for this substance.

In spite of the mild conditions of reaction (bubbling air through a 2—3 per cent solution of cellulose in Schweitzer's reagent at a temperature of 10—15° for 20—30 days), the cellulose molecule in this solution undergoes irreversible chemical changes.

After oxidation at 15° about 8 per cent of the cellulose turns into low-molecular decomposition products, soluble in water. If the oxidation is carried out at a higher temperature (30° C) about 25 per cent of the decomposition products are left in the solution after acidification.

From the glucuronic acid yield it can be calculated that under such conditions of oxidation at 15 one glucose radical out of every 30 in the cellulose chain is oxidized, half of these oxidized radicales passing into solution. Such mild oxidation was already enough to decrease the viscosity of the solution almost down to that of the solvent; besides, this oxidation was accompanied by secondary reactions as well, for such products of complete decomposition as acetic and formic acid were discovered in the solution.

After hydrolysing the oxidized cellulose with 0.2 *N* acid the solution was found to contain glucuronic, oxalic, glycolic, acetic and formic acids and formaldehyde. The content of glucuronic acid was high, in comparison with the other decomposition products in the copper-ammonia solution. The relative content of formic acid increased.

Oxidation of the products of hydrolysis made it possible to isolate arabonic acid besides the gluconic acid formed from the glucose. Erythronic acid was not found in quantities large enough for determination.

By means of special experiments it was established that no carbonic acid is formed when cellulose is oxidized in copper-ammonia solution.

Gluconic acid was determined quantitatively after Tollens and Lefevre [1] and was identified as the brucinic salt (found C—57.45, H—7.50; calculated C—57.42, H—6.33; m. p. 157—158°, published data—156—157°); oxalic and glycolic acids were identified as the calcium salts, acetic and formic acids, by qualitative reactions and by quantitative volumetric methods in the volatile acid fraction; formaldehyde was identified in the form of its dimedone derivative, m. p. 187.5° (published data—189°); arabonic and gluconic acids were identified by analysing their phenylhydrazides.

For arabonic acid: Found, per cent: N 10.94; C 51.19; H 6.68. Calculated, per cent: N 10.94; C 51.17, H 6.30; m. p. 195—196° (published data, 214°); $[\alpha]_D^{25} = +13.9°$ (published data $+14.4°$).

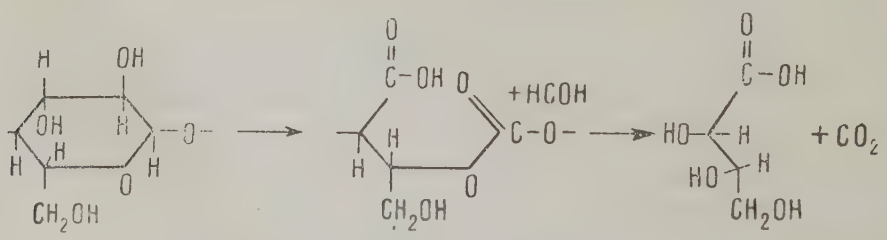
For gluconic acid: Found, per cent: N 9.92; C 47.21; H 6.37. Calculated, per cent: N 9.21: C 47.37; H 5.92; m. p. 191—192° (published data about 200°): $[\alpha]_D^{25} = +11.7°$ (published data $+12.0°$).

Decomposition Products of Cellulose Obtained by Oxidizing at 15°

	Decomposition products after oxidizing in Schweitzer's reagent	Hydrolysis products of oxidized cellulose
	Moles per number of glucose radicals in cellulose	
Formic acid	1 : 2180	1 : 192
Acetic »	1 : 127	1 : 96
Oxalic »	traces	traces
Glycolic »	none	traces
Erythronic »	none	none
Arabonic »	—	present
Gluconic »	—	present
Glucuronic »	1 : 61	1 : 61
Formaldehyde	absent	present
Acetaldehyde	absent	absent
Furfurol	present	present
Carbonic acid	absent	—

A study of the reaction products and the quantitative ratio between them offers sufficient evidence to class the oxidation process which takes place in the copper-ammonia solution as one of those reactions which begin with the oxidation of the sixth carbon atoms in the glucose radicals. This is borne out by the large amount of glucuronic acid found and the presence of arabinose and furfurol. Glucuronic acid made up 26—29 per cent of the soluble decomposition products of cellulose in the copper-ammonia solution (see table).

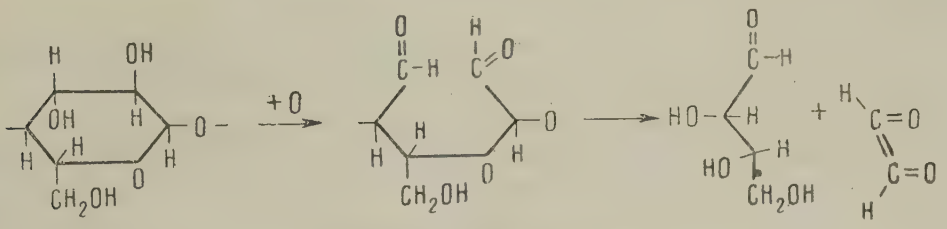
These data bear evidence against the oxidation scheme proposed by Staudinger [2] for «defective» celluloses:



Staudinger's scheme

According to this scheme, one molecule of carbon dioxide and one of formaldehyde should split off at each point of rupture of the chain. In our experiments CO_2 was not observed to have formed at all, and the quantities of formic acid formed were insignificant. The possibility of the formaldehyde disappearing from the reaction mixture before it has had time to oxidize, by condensing with the ammonia, was checked, but no such condensation products were discovered (*e. g.*, aldehyde-ammonia, amidazol or their derivatives).

The insignificant quantities of acids with two-carbon chains (oxalic and glycolic) liberated upon hydrolysis of the oxycellulose, and the absence of erythronic acid shows that in our case the splitting of the pyrane ring of the glucose radicals by oxidation, according to scheme by Jackson and Hudson ([3]), is of no great import.



Jackson and Hudson's scheme

When cellulose is depolymerized in Schweitzer's reagent, according to experiments carried out by O. P. Golova in the laboratory of the Institute of Organic Chemistry of the Academy of Sciences of the USSR, and to the data of Staudinger and Juritsch ([4]), the length of the chain decreases to 50—60—70 glucose radicals. In our experiments the quantity of glucuronic acid which passed into solution during the oxidation of cellulose corresponded to 1 molecule per 61 glucose radicals. This coincidence shows that the chain probably splits along its oxidized radicals liberating glucuronic acid. The oxidation of the hydroxyl radical of the sixth carbon atom, evidently, makes the glucoside bond unstable, as a result of which it is ruptured in alkaline solution, in which this bond is usually stable.

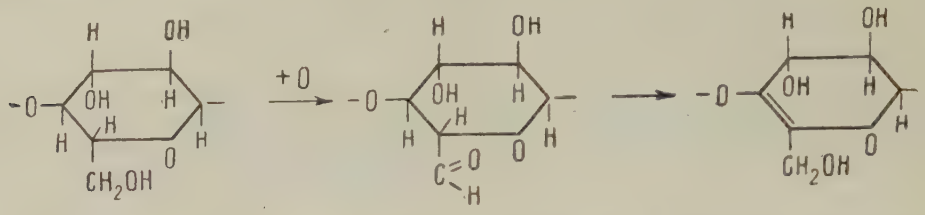
Oxidation of the end carbon decreases the stability of the glucoside bond not only in alkaline, but also in acid solution. According to our experiments, when oxidized cellulose is hydrolysed with 0.2*N* hydrochloric acid, the glucoside bonds break up 11 times more rapidly than the unchanged radicals. Thus, the stability of the glucoside bonds in cellulose is found to be impaired by oxidation, irrespective of whether the hydrolysis is of an acid or an alkaline nature. The mechanism of the reaction in the one and the other case, however, must be entirely different.

The comparative ease with which acids split off those radicals whose groups have been partly oxidized to carboxyls, has an analogy in the behaviour of polyuronic acids. These acids, which differ from the main valency chains in cellulose only in that all the sixth carbons in them wearing primary hydroxyls are substituted by carboxyl radicals, are also readily hydrolysed by acids.

But in alkaline solution neither the glucoside bond in cellulose, nor the analogous bond in polyuronic acids, break up so readily ([3]). It is hardly possible that the formation of carboxyl groups on the sixth carbon for a few solitary glucose radicals along the chain when cellulose is oxidized, could facilitate the rupture of glucoside bonds in alkaline solution.

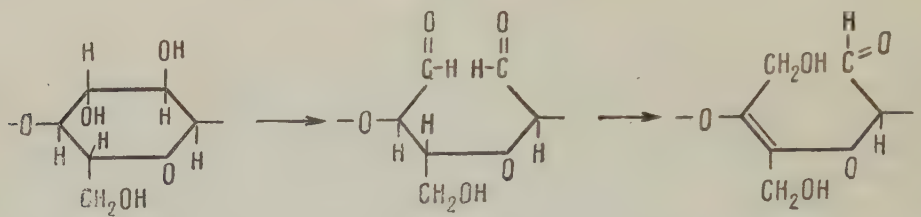
In our opinion, this impairment in stability takes place at an earlier stage of oxidation, when the alcohol radical turns into a carbonyl group. This opinion may be confirmed by examining the works of Evans and collaborators ([5,6]), who collected a great amount of material bearing evidence to the fact that the glucoside bond in glucosides of various classes hydrolyses easily in those cases where the glucoside carbon has a double bond, *e. g.* $-\overset{|}{C}=\overset{|}{C}-O-Gl$, or $N=\overset{|}{C}-O-Gl$, or where such a bond may form as a result of enolization.

By analogy with simple sugars it may be assumed that after the primary hydroxyl group in the cellulose has been oxidized to the aldehyde in alkaline solution, isomerization takes place and results in the formation of a double bond on the glucoside carbon.



In such a state the glucoside bond is unstable and hydrolyses, rupturing the cellulose chain. After this rupture the aldehyde group is oxidized further to the carboxyl.

When cellulose is oxidized with iodic acid and the pyrane ring ruptured in alkaline solution, an analogous unstable grouping may form as a result of enolization of the carbonyl on the third carbon.



This mechanism of the rupture of the glucoside bond in alkaline solution, as described above, is in full conformity with a great number of observations made for oxidized celluloses (see, for instance, the works of Davidson (7).

Hence, the formation of aldehyde groups in the oxidation of cellulose makes the structure of the latter unstable and results in rupture of the chain in alkaline solution.

Conclusions. 1. It has been established that the main trend in the reaction between atmospheric oxygen and cellulose in copper-ammonia solution is the oxidation of the hydroxyl radical on the sixth carbon atom, which results in an impairment of the stability of the glucoside bond in both acid and alkaline solution.

2. The viscosity decrease observed in copper-ammonia solutions of cellulose is due to irreversible chemical changes which take place in its macromolecule as a result of oxidation.

3. A scheme has been proposed to represent the mechanism of decomposition of the cellulose macromolecule in alkaline solution under the action of oxidizing agents.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
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PHYSICAL CHEMISTRY

THERMODYNAMIC PROPERTIES OF HIGH POLYMERS

By S. M. LIPATOV, Member of the Academy of the Bellorussian SSR

High polymers are made up of long chains having hydrophilic and lipophilic sections. Reactions that take place between these chains in the solid state may be either hydrophilic or lipophilic groups, the latter type of reaction becoming especially noticeable as the length of the hydrocarbon chain increases. That is why the action of hydrocarbon chains in high polymers should be especially well defined. According to Langmuir's principle of independence of surface action, both these types act independently, which should be taken into account in phenomena of solution, swelling, gel formation, *etc.* For the solution of simple molecules in water the heat of solution liberated $-\Delta H_p$ can be found as the sum of the heat of hydration of the polar groups q_p and the energy of their bonds in the solid gel c_p

$$\Delta u_p - \Delta H_p = q_p - c_p$$

if $q_p \gg c_p$. In the case of complex molecules, the lipophilic groups tend to prevent solution and if the lipophilic part of high polymers is long enough, limited swelling is observed instead. For the lipophilic part we may write

$$\Delta u_l = q_l - c_l + T\frac{\partial c_l}{\partial T}$$

and since in a polar medium $q_l \ll c_l$ and $c_l > 0$, then, taking into account that hydration does not depend upon the temperature within the measurable limits of the latter (Fig. 2), we have in our case for the whole molecule

$$\Delta u = \Delta u_p + \Delta u_l = (q_p - c_p) - \left(c_l - T\frac{\partial c_l}{\partial T}\right)$$

Whence it is clear that transition of a gel into a sol is possible only under the condition that a quantity of heat equivalent to the heat of «fusion» of the lipophilic bonds is applied to the system. The above equation makes it possible to explain a number of hitherto inexplicable facts:

1) limited swelling;

2) the positive temperature coefficient of swelling, when the heat effect being determined is positive;

3) the relation between the heat of swelling and the temperature, and

4) the relation between the swelling and the nature of the solvent, since a change in the nature of the solvent results in a change in the magnitude of q_l and in the ratio between q_l and c_l.

Let us examine the fundamental phenomena which accompany and cause swelling:

a) Fig. 1, which gives the equilibrium values of the volume of water absorbed by gelatin at various temperatures, shows that there exist two temperature intervals of swelling which differ pronouncedly from each other. Interval *oa* corresponds to a «frozen» state, in which the potential barrier is high and the rotational movement of the chains is greatly restricted. This interval obeys the Van't Hoff osmotic equation $\frac{ps v_1}{RT} N_2$ and is caused by the liquid being osmotically bound by the complex polydispersive structure of the gel. The second interval—*ab*—is the interval of «fusion» of the bonds between the chains in the aggregates and between the

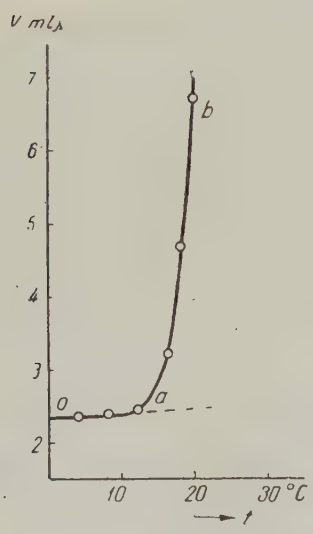

Fig. 1

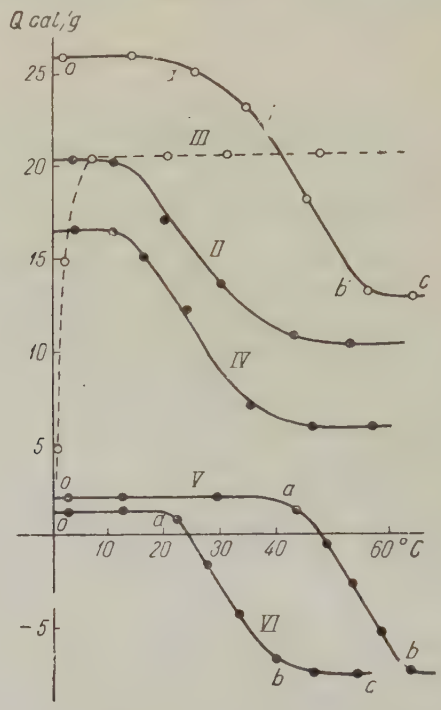

Fig. 2. *I*, agar-agar in water; *II*, gelatin in water; *III*, soluble fraction of gelatin in water; *IV*, high-nitrogen nitrocellulose in methyl alcohol; *V*, polyvinylchloride in nitrobenzene; *VI*, polyvinylchloride in dichlorethane

aggregates forming the gel. The temperature range corresponding to this interval is rather a large one, since the chains in the aggregate and the aggregates in the gel are not equivalent structurally and the energy required to deform the bonds between individual chains and aggregates is not the same. Hence each temperature has its own corresponding equilibrium state in the system. The swelling pressure in this region does not obey the Van't Hoff equation, and can be expressed by the following equation

$$p_s = aTn_T = aTn_0 e^{\Delta H/RT}$$

where a is a constant; n_T, the number of particles at the investigated temperature T, and n_0, the number of particles at the initial temperature. Since there is an equilibrium ($-\Delta F = 0$) for each temperature, at each transition point

$$\Delta S = \Delta H/T$$

and if the temperature interval is not very large, we have for the entire region

$$\Delta S = \Sigma \Delta H/\overline{T}$$

From the data given in Fig. 1 it has been computed that $\Delta H = 25\ 000$ cal.mol*. Mayer ([1]) found the value $\Delta H = 21\ 000$ cal./mol on the basis of the heat of jellification, and Kuntzel ([2]) calorimetrically determined the heat of jellification to be equal to 12.5 cal./mol.

b) The influence of the temperature on the heat of swelling had not been investigated until now. Hence it was held that the heat effect does not depend upon the temperature and that this accounts for the immeasurably small energy of the chain bond. A detailed investigation carried out in our laboratory by S. I. Meyerson showed that this is not true. The heat of swelling-temperature curves given in Fig. 2 indicate that just as in the first case (Fig. 1) there is a broad temperature interval *oa* in which the heat effect does not depend upon the temperature; in this interval $q_p \gg c_p$ and $c_l = \text{const}$.

When the temperature has risen sufficiently to deform the bonds, the heat effect declines abruptly (region *ab*), and after complete rupture of the bonds it again becomes independent of the temperature (region *bc*). Fig. 2 shows that the heat jump for gelatin (curve *II*) equals 9.7 and for agar (curve *I*) 13 cal./g, these figures being close to Kuntzel's data. It should be noted that the highly dispersed fraction of gelatin which is soluble in water at as low a temperature as 5°C and contains the same number of NH_2- and COOH-groups as gelatin, gives a heat effect (curve *III*) close to that for the latter, but which is independent of the temperature within the interval between 5 and 60 C. This proves that within the interval 5—20° C the heat of swelling of gelatin is determined only by the reaction between the NH_2-, NH—CO- and COOH-groups and water, and that only at temperatures above 20° do the bonds between the chains begin to rupture. This proves also that within the temperature range 5—60°C investigated by us the heat of hydration, and, consequently, its magnitude also, do not depend upon the temperature; hence the decrease in the heat effect is due only to the rupture of the bonds between chains, whose interaction energy is sufficiently high. This interaction results in the formation of aggregates both in the gel and in the solution. This conclusion may be arrived at from another angle of approach as well.

c) Considerable influence of the temperature on the volume of liquid absorbed by a jelly while swelling ($p_s = \text{const}$) should become manifest in investigating the swelling pressure ($v = \text{const}$), and also in investigating the osmotic pressure of sols. This method of investigating high polymers was extensively employed by Schultz([3]), Mayer ([4]), Eyring ([5]) and others. They worked, however, in a very narrow range of temperatures. We have investigated the osmotic properties of high polymers in a broad range of temperatures right up to the «critical temperature» of solution. This has made it possible to compute all the thermodynamic functions: ΔF, ΔS and ΔH. The change in free energy can be computed directly from the osmotic data

$$\Delta F_1 = -(p_s v_1)_{T_1 N}; \quad \Delta F_1 = -(p_o v_1)_{T_1 N}$$

where p_s is the swelling pressure, p_o, the osmotic pressure, and v_1, the molar volume of the solvent. The change in entropy was determined from experimental data by means of the equation

$$\Delta S_1^{\text{exp}} = \left(\frac{\partial \Delta F}{\partial T}\right)_v$$

and the ideal values of ΔS_1, from the molecular weights, determined osmotically

$$\Delta S_1^{\text{id}} = -R \ln(1 - N_2)$$

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* The measurements were made by V. N. Balabanova.

where N_2 is the molar fraction of the second component. Both these quantities are connected by the equation

$$\Delta F_1 = \Delta H_1 - T\Delta S_1$$

from which ΔH_1 can be computed.

Fig. 3 gives the values of $p_o = f(T)$. Just as in the two examples analysed above, we have three regions: region *oa* exactly obeys the Van't Hoff equation:

$$\frac{p_o v_1}{RT} = N_2$$

so does region *bc*, but at different values of N_2. For region *oa* $M = 27\,000$ and at a gelatin concentration of 5 g/litre, $N_2 = 4 \cdot 10^{-6}$, whence

$$T\Delta S_1^{id} = -RT \lg (1 - N_2) = 0.0023$$

and

$$T\Delta S_1^{exp} = T\left(\frac{\partial \Delta F}{\partial T}\right)_v = 0.0024$$

Hence, in region *oa* the binding energy of the particles is so great that each aggregate acts as an independent kinetic unit, moreover $\Delta H_1 = 0$ and the solution behaves

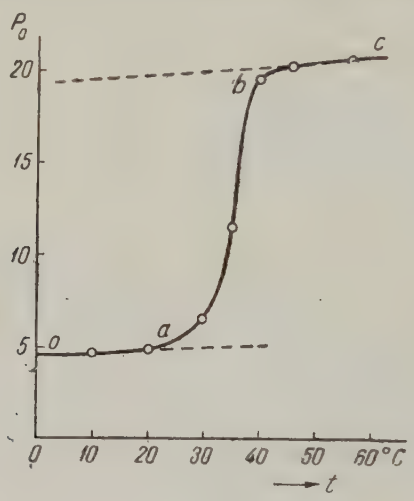

Fig. 3

as if it were ideal. In region *bc*, $M = 10\,000$ and for the same concentration we get

$$T\Delta S_1^{exp} = 0.0066;\ T\Delta S_1^{id} = 0.0065;\ \Delta H_1 = 0$$

In region *bc* we have a molecular system. Region *ab* is a region of «fusion» of the aggregates; it is characterized by $\Delta F_1 \approx 0$; $\Delta S_1 \approx \Delta H/T$ and the value $T\Delta S_1 = 0.36$, *i. e.*, is 60—100 times greater than the above value. Hence we have

$$(T\Delta S_1)_{ab} \gg (T\Delta S)_{bc} > (T\Delta S_1)_{oa}$$

Mayer's conclusion that the entropy of a system with long chains consderably exceeds its ideal value ([6]) cannot be generalized, the more so since he determined the molecular weight of the ideal solution incorrectly. It should be noted that Hagger and Van der Wik ([7]), while working on dilute solutions of acetyl-cellulose in tetrachlorethane, obtained experimental values for the entropy, which coincided with the ideal values. The nature of the solvent has a considerable influence on ex-

perimental results. Thus, dilute solutions of rubber in chloroform, where $M=9\,000$, give the following results:

g/litre	p_0 mm Hg		$T\Delta S_1^{exp}$	$T\Delta S_1^{id}$
	10°	20°		
2	50	53	0.0104	0.0104
5	130	139	0.0291	0.0261
10	270	286	0.0580	0.0522

In other solvents $\Delta H_1>0$ and the system deviates from the ideal owing to aggregation. This is easily established by studying the temperature coefficients of the osmotic pressure. Positive temperature coefficients correspond to positive deviations from Raoult's law and $p_{exp}<p_{id}$. At the same time Mayer used the assumption that $p_{exp}>p_{id}$ as his initial point, basing his deliberations on the equation

$$p_{exp}=\frac{RT}{M}c_2+\alpha c_2^2=p_{id}+\alpha c_2^2$$

where the term αc_2^2 represents supplementary entropy action due to the internal mobility of the chains. This equation may be used, when applicable, to highly dilute solutions as well, *i. e.*, when Van't Hoff's limit law does not hold over the entire range of the curve $p_{exp}=f(c_2)$, and when the relation between the osmotic pressure and the temperature is normal. For solutions of high polymers, the following is observed, depending upon the nature of the solvent: 1) dilute solutions obey Van't Hoff's limit law, so that $p/c_2=$const (rubber—chloroform); 2) high temperature coefficients for the osmotic pressure when $p_{exp}\ll p_{id}$ (gelatin, agar, nitrocellulose in benzyl alcohol, *etc.*). In a number of cases the additional entropy effect is practically immeasurable, since it is concealed by the complex interaction of the chains both with one another (aggregation, association) and with the molecules of the solvent (solvation). All this may affect essentially the shape of the chains and the internal mobility of their individual parts.

Conclusions. 1. An investigation of the temperature coefficient of swelling and of the relation between the heat of swelling and the temperature has shown that the binding energy of a chain is of the order of 10^3—10^4 cal./mol and cannot be assumed to be infinitely small.

2. Solutions of high polymers most frequently show positive deviations from Rayleigh's law and not negative deviations, as sssumed by Mayer and others.

3. When N_2 is correctly computed, the experimental and ideal values of the entropy coincide if $\Delta H_1=0$ (positive deviations are absent).

4. In the temperature interval corresponding to the deformation of the bonds between the chains, $\Delta S_1\approx\Delta H/T$, *i. e.* «fusion» of the bonds takes place.

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Received
24.V.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

GEOCHEMISTRY

ON THE CONTENT OF LEAD IN PLANTS

By S. I. SINIAKOVA

(Communicated by W. I. Vernadsky, Member of the Academy, 20. XI. 1944)

Malaguti, Durocher and Sarzeaud (1) were the first to discover the presence of lead in see algae, in 1849. Later on Forschhammer (2) pointed out the presence of this element in various species of woody plants. Hattensauer (3), analysing the ash of *Mollinia abtissima* Link, found it to contain 2.041 per cent of PbO. Traces of lead were reported for various plant species by Vogtherr (4). The ash of *Agrostis*, *Calamagrostis*, analysed by Emmerling and Kolkwitz (5), was found to contain up to 3.56 per cent of lead. During the second and third decades of this century there appeared a number of qualitative and semiquantitative data concerning the content of lead in various plants, determined mainly with the aid of the spectroscopic method (6-9).

All these determinations were of qualitative nature, while the quantitative data obtained seemed to be insufficiently reliable. And it was not until quite recently that number of plant species, serving as raw material for various food products, were subjected to a more detailed and systematic investigation, on account of the toxicity of lead taken with food, which formed the subject of independent studies.

Thus, Lynch, Donell, Heydwood and Waite (10) discovered in various small fruits $6 \cdot 10^{-5} - 1.9 \cdot 10^{-4}$ per cent of lead; Kehoe, Thammann and Cholak (11) determined a content of $3 \cdot 10^{-5} - 9.4 \cdot 10^{-5}$ per cent of lead in various food products. Similar data were also reported by Monier-Williams (12). Finally, Ivanov (13) and Kogan (14), who analysed different food-products (grain, *etc.*), determined a lead content ranging from $n \cdot 10^{-5}$ to $n \cdot 10^{-6}$ per cent.

In the work here reported representatives of several families of terrestrial plants and a number of sea algae were investigated for their Pb content; the material was collected during the expeditions of the W. I. Vernadsky Laboratory for Geochemical Problems in different parts of this country. The method of analysis applied was as follows: a 100 g sample of air dry plant material was ashed in a muffle at a temperature not exceeding 500° C. The ash was passed to solution. Silicic acid was separated according to the routine method, the lead was extracted with dithizol, and determined with the aid of the polarographic method. The data obtained are tabulated below.

The results reported permit of the following conclusions. Lead is not accumulated by terrestrial plants. Its content in the latter fluctuates from $n \cdot 10^{-6}$ to $n \cdot 10^{-5}$ per cent to live weight. The content of lead in the soils on which these plants grow was found to be equal to $n \cdot 10^{-3}$ per cent. The content of lead in the ash of terrestrial plants ranges from $n \cdot 10^{-5}$ to $n \cdot 10^{-4}$; this is in good agreement with the amount of lead we were able to extract from the soils with the aid of different acids, which was also found to be equal to $n \cdot 10^{-4}$ per cent. Among the plants investigated, the highest in Pb are the representatives of the family *Gramineae*. In individual re-

Family	Plant species	Place of collecting and year	Pb content, in %		
			to live weight	to air-dry matter	to ash
Gramineae	*Panicum Crusgalli* L.	Staroselye, 1936 . .	$1.4 \cdot 10^{-5}$	$5.1 \cdot 10^{-5}$	$4.55 \cdot 10^{-4}$
»	*Calamagrostis epigeios* Roth.	» 1936 . .	$1.2 \cdot 10^{-5}$	$2.0 \cdot 10^{-5}$	$2.30 \cdot 10^{-4}$
»	*Agrostis vulgaris* With.	Zvenigorod, 1936 .	$0.3 \cdot 10^{-5}$	$0.8 \cdot 10^{-5}$	—
»	*Phleum pratense* L.	» 1936 .	$0.7 \cdot 10^{-5}$	$1.2 \cdot 10^{-5}$	$1.70 \cdot 10^{-4}$
»	*Stipa capillata* L.	St. Talovaya . . .	$0.1 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	$0.45 \cdot 10^{-4}$
Compositae	*Centaurea Jacea* L.	St. Bolshevo, 1935	$0.2 \cdot 10^{-5}$	$0.5 \cdot 10^{-5}$	—
»	*Hieracium umbellatum* L.	Staroselye, 1936 . .	$0.05 \cdot 10^{-5}$	$0.1 \cdot 10^{-5}$	$0.22 \cdot 11^{-4}$
Liliaceae	*Veratrum nigrum* L.	At south from lake Yamkun	$0.06 \cdot 10^{-5}$	$0.6 \cdot 10^{-5}$	$0.80 \cdot 10^{-4}$
Papilionaceae	*Medicago falcata* L.	Staroselye, 1936 . .	0	0	0
Caryophyllaceae	*Silene latifolia* Britte	Zvenigorod, 1936 .	0	0	0
»	*Setaria viridis*	» 1936 .	0	0	0
»	*Bangia fusuporia*	» 1936 .	0	0	0
Laminariaceae	*Laminaria digitata*	Biological Station of Murmansk			
		Zelentzy, 1938 . .	$0.04 \cdot 10^{-5}$	$0.2 \cdot 10^{-5}$	$0.8 \cdot 10^{-5}$
»	*Phyllaria dermat.*	» 1938 . .	$0.3 \cdot 10^{-5}$	$2.7 \cdot 10^{-5}$	$7.6 \cdot 10^{-5}$
»	*Alaria esculenta*	» 1938 . .	$0.1 \cdot 10^{-5}$	$0.56 \cdot 10^{-5}$	$2.4 \cdot 10^{-5}$
Fucaceae	*Fucus serratus*	» 1938 . .	$0.08 \cdot 10^{-5}$	$0.31 \cdot 10^{-5}$	$1.5 \cdot 10^{-5}$
»	*Ascophyllum nodosum*	» 1938 . .	$0.2 \cdot 10^{-5}$	$0.8 \cdot 10^{-5}$	$4.1 \cdot 10^{-5}$
Potamogetonaceae	*Ruppia maritima* L.	» 1938 . .	0	0	0
»	*Zostera nana* Roth.	» 1938 . .	0	0	0
Ulvaceae	*Ulva lactuca Algae*	New Haven Conn .	0	0	0

presentatives of this family the content of lead is found to fluctuate within a rather considerably range. No lead at all was discovered in representatives of the families *Caryophyllaceae* and *Papilionaceae*. Sea algae accumulate lead to an insignificant degree only, for, according to the data available, the content of this element in sea water is about $4 \cdot 10^{-7}$ per cent ([15]). Sea grass was found to contain no lead. Sea algae are rich in cadmium which they contain at a ratio ten times exceeding that of lead.

Received
20.XI.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

BIOCHEMISTRY

CERTAIN BIOCHEMICAL FEATURES PECULIAR TO THE FUNGUS *VERTICILIUM DAHLIAE* KLEBAHN

By V. A. ANISIMOVA

(Communicated by A. A. Richter, Member of the Academy, 24. I. 1945)

The object of the studies here reported was to decipher the biochemical nature of *Verticillium dahliae* Klebahn with reference to a wilt disease affecting cotton in Central Asia.

Verticillium dahliae Klebahn, which is a facultative parasite, is the causative agent of tracheomycosis wilt of American cotton varieties. Penetrating into the plant *via* roots, the fungus causes severe disturbances of all the internal processes and has in most cases a lethal effect. The author undertook to make out the way in which the parasite utilizes the carbohydrate and nitrogen reserves of the plant, and also the activity of carbohydrates and proteases both in a medium that served previously for the culture of this fungus, and in its mycelium.

As material use was made of a pure culture of *Verticillium dahliae* Klebahn isolated from cotton plants. The fungus was grown in glass flasks, upon liquid mineral medium, originally of the following composition: $MgSO_4$—0.03 per cent, K_3HPO_4—0.035 per cent, $Ca(Na_3)_2$—0.35 per cent, $FeSO_4$—traces. In accordance with the requirements of the experiment, the organic forms of nitrogen and carbohydrate were introduced in the medium in a form analogous to that to be tested. After being sterilized, the flasks containing 100 ml of the nutrient solution each, were infected with the fungus culture and kept in a thermostat at 19—27°C until analysed.

To judge of the degree to which the fungus consumed the sugars and nitrogen of the medium, the latter was analysed for carbohydrates and nitrogen. Each time the contents of different flasks were taken for analysis. Sugar was determined after Issekutz, total nitrogen, after Kjeldahl, aminonitrogen, after Sörensen. The activity of the enzymes was determined by the routine method.

Among the sources of carbohydrate nutrition, under test were galactose, glycose, fructose, saccharose, starch and cellulose (absorbent cotton, filtering paper); among the sources of nitrogen nutrition, peptone and edestine were tested.

Below are tabulated data on the utilization by *Verticillium* of simple sugars.

As may be seen from table 1, all the sugars are well consumed by the fungus, glycose is, however, the best assimilated. Thus, for example, by the 15th day of growth, up to 90 per cent of glycose are used up, whereas galactose and fructose are consumed to no more than 60 per cent.

As shown by our experiments, saccharose is not consumed by *Verticillium* until its hydrolysis to monoses has taken place (Table 2). It should be pointed out that the fungus is endowed with a very high inversion ability with respect to this sugar: it is unable to consume the total product of inversion which accumulates as a result of this process.

Table 1

Age of fungus, in days	Galactose		Glucose		Fructose	
	sugar in mg per 100 ml	pH of solution	sugar in mg per 100 ml	pH of solution	sugar in mg per 100 ml	pH of solution
0	288.5	5.4	366.0	5.4	357.0	5.4
4	273.0	5.73	365.7	5.43	300.0	5.57
6	218.5	—	244.2	5.48	239.0	—
9	199 4	6.34	252.3*	5.5	142.0	7.1
13	137.0	7.13	86.5	6.34	157.5*	7.23
15	113.0	—	28.8	—	128.9	—
20	80.8	7.29	11.2	6.98	—	7.65
23	—	—	8.55	—	72.9	—
27	39.3	7.58	21.1*	7.73	23.2	8.0
32	36.6	—	7.5	—	24.3*	—
38	28.7	—	8.4*	—	24.4*	—

* The increased carbohydrate content was due to the fact that in each separate case the contents of a new flask was taken for analysis. Allowance being made for the variations in the development of the mycelium, deviations in the intensity of utilization of the substratum were quite permissible, too.

Table 2

Age of fungus, in days	Amount of sugar per 100 ml of solution			Dry weight of fungus, in mg
	monoses, in mg	saccharose, in mg	total sugar expressed in saccharose*	
0	27.3	357.7	383.6	—
9	40.5	236.1	315.1	55.3
14	19.4	96.7	115.1	—
17	28.8	81.2	108.6	—
19	27.0	19.35	45.7	112.5
21	21.2	33.1	53.2	—
23	8.0	No sugar	7.6	117.5
24	No sugar			

* When summing up, the amount of monoses was multiplied by 0.95.

Table 3

Age of fungus, in days	Sugar in mg per 100 ml of solution			pH of solution	Dry weight of fungus, in mg
	monoses	saccharose	starch		
0	0	0	330.8	—	—
3	0	8.74	241.9	—	—
6	20.8	10.3	218.97	—	—
9	44.3	2.44	96.7	6.93	—
13	50.1	0	71.1	—	82
17	35.6	0.46	45.0	7.3	—
21	13.2	0.95	14.7	—	—
24	4.75	0	3.8	—	129.6
28	5.5	1.74	2.99	7.52	135.4
35	6.9	0	2.3	7.8	—

As may be seen from the results of our experiments tabulated above, the starch is readily assimilated by *Verticillium*: by the 24th day of growth, up to 99 per cent of the starch were found to be used up by the fungus. The presence of saccharose in the solution, especially during the first days of growth, should be taken to mean that the hydrolysis of starch is effectuated by the fungus' enzymes through the saccharose, which is in agreement with the latest data by Lvov and Fichtenholz ([3]).

The enzymes which are inherent in *Verticillium dahliae* are also capable to hydrolyse cellulose* to monoses: during this process a number of intermediate products are formed. When analysing the medium in which the fungus was grown, the hygroscopic cotton was found to bear traces of monoses, disaccharoses and dextrines (Table 4). On account of the circumstance that it is no easy matter to isolate the fungus from the substratum, we contented ourselves with a mere record of the residual cellulose. Towards the 52nd day of cultivation the quantity of the cellulose decomposed made up 36 per cent.

It should be pointed out that upon cellulose, especially when grown upon filtering paper, the fungus was found to develop at a somewhat lower rate than on other carbohydrates.

Table 4

Cellulose, in mg	Age of fungus, in days	Carbohydrates in mg per 100 ml of solution			Weight of residual cellulose, in mg
		monoses	disaccharoses	dextrine	
338	0	—	—	—	—
	17	4.69	2.73	0	—
	19	7.91	2.65	0.54	—
	24	8.14	1.39	0.27	—
	31	9.35	1.28	2.79	—
	44	7.46	1.46	2.87	—
	52	6.31	0	2.86	215.2
	60	6.6	0.6	2.91	—

The results of our experiments on the assimilation of peptone are presented in Table 5.

Table 5

Age of fnngus, in days	Amount of nitrogen per 100 ml of solution, in mg	
	total nitrogen	amino-nitrogen
0	158.4	13.7
7	134.4	22.7
12	—	39.3
20	131.6	52.3
27	130.9	58.7
36	118.3	56.4
42	114.1	60.9

The figures of Table 5 point to an intense splitting of peptone accompanying the growth of the fungus. As early as towards the 7th day of cultivation, the medium is found to show an increment of amino-nitrogen, while total nitrogen has been found to decrease.

* Oknina ([1]) failed to discover any cellulose.

When grown on peptone, the fungus showed an intense development since the very first days of cultivation, so that a solid film was usually found to be formed over the liquid by the 25th day of growth.

The fermentative hydrolysis of edestine takes a much slower course, which seems to be due to this protein being sparingly soluble in water*. Only traces of amino-nitrogen could be discovered in the medium in the process of growth of the fungus when this protein was used as substratum. Alongside of a lower rate of assimilation of edestine, the mode of accumulation of the vegetative mass differs greatly: the fungus was found to spread over the liquid in the form of a loose mucous.

The investigation of the enzymes of *Verticillium* has shown this fungus to possess active endo- and ecto-enzymes.

The study of these enzymes is of interest in connexion with its harmful influence upon the host plant, which was also investigated. The more active the enzymes, the more complete is the hydrolysis of the reserve substances brought about by the fungus in the tissues of the host plant, and the higher the harmful activity of the parasite. The results of our experiments on the activity of amylase, invertase and protease are represented in Table 6, the control being put at 100.

Table 6

	Amylase	Invertase	Protease
Medium	171	121	241
Extract from mycelium	688	485	142

Conclusions. The fungus *Verticillium dahliae* Klebahn assimilates along with soluble forms of organic matter, sugars and amino-acids, also such compounds as polysaccharids (starch and cellulose) and proteins (edestin and peptone), which are preliminarily split to soluble forms by the ecto-enzymes of the fungus. The splitting by the parasite of high molecular organic substances of the host plant to a soluble form does not only interfere with the normal metabolism of the host, but leads inevitably to carbohydrate and protein starvation. Moreover, the enzyme of the fungus, the cellulase, is very likely to be able at a definite stage of the fungus' development to hydrolyse the cellulose of the vascular walls, thus disturbing the integrity of the conveying system.

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Received
24.I.1945.

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* According to data by Oknina ([1]), *Verticillium dahliae* Klebahn gave but a poor increment when grown upon protein compounds (peptone, gelatine) while it showed an intense growth on a medium containing sparingly soluble protein, *viz.* casein.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

CHEMICAL TECHNOLOGY

EFFECT OF COAL ASH ENRICHED WITH LIME UPON HYDRATION OF INSOLUBLE ANHYDRITE

By P. P. BUDNIKOV, Corresponding Member of the Academy, and J. M. BUTT

Coal ash can be made into binding material either simply by adding lime to it or by burning the coal in the form of powder, mixed with limestone. The latter method of coal ash enrichment with lime has been worked out in detail by E. Z. Udovich, O. A. Gerschberg, N. D. Kevesh and E. D. Lebedev. To the product obtained in this way they have given the name of TEZ-cement (1). It is a close mixture of a fine dispersed ash, free lime and a certain amount of silicates, aluminates and ferrites of calcium formed upon burning air-coal-lime mixture in power station furnaces as a result of interaction between the lime and the components of the ash.

TEZ-cement contains 32 to 48 per cent of lime and 7.5 to 25 per cent of alumina, the ratio of free lime varying within 10—20 per cent. This wide variation of the content of lime results in a corresponding variation of the volume, which is a serious drawback with the TEZ-cement and greatly restrains the range of its application.

The present authors have suggested the production of a two-component cement composed of insoluble anhydrite or some other modification of calcium sulphate and of coal ash enriched with lime. Enrichment with lime can be carried out before burning the coal as well as after. In combination these two materials must be very effective, since lime in an active state acts as a catalyst upon the hydration of insoluble anhydrite, and the different modifications of calcium sulphate stabilize the volume of the solidifying lime (2) and eliminate or sharply decrease variations in the volume of the lime-ash combinations.

In the process of hydration of the two-component cement, which we describe for brevity as AT-cement, and which consists of insoluble anhydrite and TEZ-cement, silicates, aluminates and ferrites of calcium undergo transformation into the respective hydrates, which are the hydraulic components of the AT-cement. These hydrated components which are in a colloidal state, envelope the particles of the insoluble anhydrite and render the latter hydraulic.

Some silicon that has failed to react in the process of coal burning will be present in the ash in a finely dispersed active state and will react on hydration with free lime yielding hydrosilicates of calcium along with those hydrosilicates which are obtained upon the hydration of the bicalcium silicate formed in the process of burning.

These reactions proceed according to the following equations

$$2Ca(OH)_2 + SiO_2 + nH_2O = 2CaO \cdot SiO_2 \cdot nH_2O$$
$$2CaO \cdot SiO_2 + nH_2O = 2CaO \cdot SiO_2 \cdot nH_2O$$

Calcium aluminates while reacting with calcium sulphate in the presence of lime will form sulphoaluminates of calcium alongside of hydroaluminates (3).

Calcium ferrites will behave similarly yielding sulphoferrites of calcium instead of sulphoaluminates.

The presence of CaS in TEZ-cement brings on the formation in the solidifying AT-cement of calcium hydrosulphate and calcium hydroxide, which are both highly active at the time they are forming.

Free lime, as was mentioned above, is a catalyst in relation to anhydrite, while the residual silicon and the other components of the ash that have taken no part in the reaction, fill the pores, thus making the cement denser and increasing its strength and hydraulic qualities. Thus, from theoretical data we appear to have in the AT-cement a new kind of binding material of great strength and high hydraulicity.

These theoretical conclusions were checked by experiments in which the production of AT-cement was tried and its properties studied. As initial materials bi-aqueous gypsum from Ust-Kama deposit, TEZ-cement and TEZ-ash were used. The chemical analyses of the initial materials (in per cent) are given in Table 1.

Table 1

Components	Gypsum	TEZ-cement	TEZ-ash
SiO_2	0.04	19.82	38.18
Al_2O_3	0.07	8.00	25.89
Fe_2O_3		9.49	11.19
CaO	31.84	41.91	19.87
MgO	0.08	1.28	1.33
SO_3	46.00	7.12	1.42
Loss on ignition	—	12.81	2.06
Bound water	21.29	—	—
CO_2	0.77	—	—

The bi-aqueous gypsum was burnt in a laboratory electric furnace at 700°C and then reduced to a powder leaving a residue of 6 to 8 per cent on a screen with 4900 openings per square centimetre. The material thus obtained was blended with TEZ-cement or with TEZ-ash in different proportions, after which the binding qualities of the cements were tested.

With small samples the mechanical strength was determined by preparing plastic solutions without sand (1 : 0), keeping them for three days in a moist medium and drying at 60° C to constant weight. Water-cement ratio, 0.36. The results of the tests are given in Table 2.

Table 2

Mechanical Strength of Cements (in kg/cm²)

Composition, per cent				Compression	Bending
anhydrite	TEZ-cement	TEZ-ash	lime		
90	10	—	—	375	87.0
85	15	—	—	370	78.0
80	20	—	—	306	61.5
70	30	—	—	293	75.3
59	50	—	—	275	66.6
20	80	—	—	168	48.5
10	90	—	—	66	30.5
0	100	—	—	60	37.2
89	—	10	1	213	65.1
83.5	—	15	1.5	263	68.7

Cement of the optimal qualities was tested in standard plastic samples (1 : 0) under the same conditions of preliminary treatment. Its optimal compression strength was found to be 331 kg/cm²; optimal tensile strength 24.5 kg/cm². The data show that standard samples differ little in compression strength from small samples.

Tests of standard rammed samples with normal sand 1 : 3 under the same conditions of solidifying gave the following results: ultimate compression strength, 364 kg/cm²; ultimate tensile strength, 31.1 kg/cm².

Uniformity in the change of volume was tested by keeping samples shaped into cakes at 120 C, which is consistent with the adopted standard for anhydrite cement. Other cakes were put into cold water, kept in steam or boiled, according to the standards adopted for hydraulic binding materials. The water-cement ratio in all cases was 0.36. All the cements tested were found to withstand drying at 120° C. But in water tests and in hot tests the TEZ-cement without anhydrite and the samples containing 90 per cent of TEZ-cement failed. These results indicate, on the one hand, that TEZ-cement and TEZ-ash impart hydraulicity to anhydrite cement, and on the other, that the negative quality of the TEZ-cement, *viz.* its failure to change uniformly in volume, is made good by the addition of anhydrite cement.

Setting time tests have shown that in the case of the anhydrite cement containing a small proportion of TEZ-cement (up to 30 per cent) setting begins after 30 to 50 minutes and ends after 2 to 2.5 hours. The process is retarded as the content of TEZ-cement is increased and the content of anhydrite diminished. In pure TEZ-cement setting begins after 3 h. 30 m. and ends after 5 h. 20 m.

Conclusions. 1. Coal-ash enriched with lime accelerates the hydration of insoluble anhydrite and increases the strength and water resistance of the cement.

2. Anhydrite when added to TEZ-cement greatly improves the qualities of the latter and prevents irregular variations of volume.

3. Anhydrite combined with lime-enriched ash yields a new kind of binding material, which we have given the name of AT-cement and which enjoys many good qualities.

4. AT-cement can find wide application in new construction as well as in rehabilitation work, and is obtainable from local raw materials and from burned fuel waste products.

Received
29. IV. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

GEOLOGY

ON THE SPAR-BEARING PROVINCE OF THE SIBERIAN TRAP REGION

By G. MOÖR

(Communicated by V. A. Obruchev, Member of the Academy, 15. X. 1944)

The deposits of iceland spar which are genetically related to the lava sheets of the formation of Siberian traps and confined to the middle course of the Nizhnaya Tunguska River, have of late advanced to the first place among the accumulations of this mineral in this country. They compare favourably with the deposits occurring in limestones and dolomites, from the point of view of regional distribution of the optical material of spar-bearing occurrences, their magnitude, the recovery of iceland spar in proportion to the raw material and the quality of the spar. The Tunguska deposits considerably exceed the accumulations of spar associated with sedimentary rocks. As well known, the latter are mostly rather small, have but small reserves and vary greatly with regard to the content of iceland spar, which is not infrequently of inferior quality.

All this considerably reduces the value of the accumulations of iceland spar associated with carbonate rocks. Within the recent years attention has, therefore, been attached to the search and prospecting for iceland spar deposits associated with lava sheets and, in the first place, with effusive traps which are widely developed in the basin of the middle course of the Nizhnaya Tunguska River and along some of its tributaries.

The effusive members of the Siberian trap formation occupy a vast territory within the northern parts of the Siberian platform. This territory is located in the basin of the Nizhnaya Tunguska, Kotuya, Kureika, Kheta and upper course of the Vilui rivers. The area occupied by lavas attains 350 000 sq. km according to preliminary estimation; not included here is a vast territory in the basin of the Vilui River in its upper and middle course, since the nature of the trap bodies there is not quite clear; the possibility is not excluded that in this case bedded deposits of traps, brought to the day's surface by erosion, have been described as sheets.

The lava field of the Siberian platform is composed of numerous sheets of traps, alternating in a baroque manner and forming on the whole a thick effusive series; structurally, the lava field presents an integer geological unit.

The lava series attains its greatest thickness in the central parts of the vast field of lavas within the area of the Kotuya divide, Nizhnaya Tunguska, Kureika and Kheta rivers, where are situated the highest points recorded at present over the territory between the Yenissey and the Lena rivers. Towards the peripheral parts of the lava field the thickness of the lava series gradually decreases, reaching only 350—450 m along its borders, in particular in the valley of the Nizhnaya Tunguska River. The thickness of individual sheets varies from a few metres to several dozens of metres. However, of most frequent occurrence are sheets measur-

ing 8—15 m, which, in general, approaches closely enough the thicknesses of the sheets and flows of most of the largest lava fields of the Earth. Judging by the fact that even in some large exposures in the valley of the middle course of the Kotuya and the Nizhnaya Tunguska rivers the number of sheets is often found to attain ten and even more, their number must be very considerable*.

At the present time the boundaries of the lava field have been more or less definitely fixed only for its southern marginal part, within which passes the valley of the middle course of the Nizhnaya Tunguska River. As to the other areas of the lava field, with the present state of knowledge, its boundaries are traced rather arbitrarily.

As regards the age, the lava series refers to the Lower Mesozoic. The outflows of lava were mainly confined to the Lower and partly to the Middle Triassic; it is hardly probable that they occurred also during the Upper Triassic and still less the early Jurassic. This is evidenced by the data of G. Moör and B. Tkachenko, according to which the lava series overlies the deposits belonging to the Korvunchana series (upper part of the section of the Angara series), containing an abundant flora of an early Mesozoic habit and faunistic remains (*Estheriae*, ostracods, placoderms), indicating the uppermost horizons of the Permian or lower horizons of the Triassic.

The lava sheets confined to the lower horizons of the lava series occasionally display in their attitude disturbances of an appreciable magnitude. As shown by G. Moör, they form local folded structures of the type of short folds, with a strike approaching the meridional one with angles of dip in the limbs up to 45°. The superjacent lava sheets have a horizontal attitude, transgressively overlying the lower horizons of lava sheets. Among the latter in layers of tufaceous rocks *Estheriae* pointing to the lower horizons of the Lower Triassic (Vetluga stage) have been found.

This indicates the age of the fold disturbances, on the one hand, and the time of the outflow of the superjacent sheets, on the other.

Within the lava field of the Siberian platform the spar occurrences known at the present time have been confined to the southern border of the lava area, mainly to the basin of the middle course of the Nizhnaya Tunguska River. As to the central and northern parts of the lava field, far removed from the valley of this river, no findings of iceland spar have been reported here; this should be explained in the first place by the fact that the lava field as a whole and its parts located in the basin of the Kotuya, Kheta and Kureika rivers, in particular, have been quite insufficiently explored. It should be borne in mind that the latter areas were crossed only by one geological route—by I. Tolmachev and O. Baklund—as early as 1905; the materials of this survey largely remain unstudied still.

Spar occurrences are widespread within the southern marginal part of the lava field. Iceland spar findings are known in the middle course of the Nizhnaya Tunguska River over a considerable portion of the valley adjoining the mouth of the Kochechuma River, and along several tributaries, part of which are rather small—the Gonchak, Poledjikit, Goncha, *etc.*, while others are larger—the Vivi, Yambukan, Kochechuma with tributaries (Tembenchi, Turu, *etc.*), Ilimpeya, *etc.*

The accumulations of iceland spar which are of economic value, as far as it can be judged from the author's investigations carried out during recent years, are subordinate to the lower horizons of the lava series exposed in the middle course of the Nizhnaya Tunguska River and directly overlying the tufaceous deposits. In these portions of the section of the lava series numerous manifestations of hydrothermal activity are recorded, which have caused here the appearance of pockets, veins and various-shaped mineralized areas composed of calcite, including its optically transparent varieties—iceland spar, chalcedony, quartz and zeolites, among which mordenite is the first to be mentioned.

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* In this connexion it should be borne in mind that T. Torodsen distinguished up to 87 individual sheets among the young (old Quaternary and recent) effusions of basalts in one of the regions of Iceland.

As to the higher horizons of the lava series from observations of itinerary survey in the central parts of the lava field, in the upper course of the Kochechuma and Kotuikana, they are characterized in general by a greater simplicity of structure; neither layers of tufaceous rocks of appreciable thickness, nor peculiar agglomerate lavas, occurring in the lower part of the section of the lava series, have been discovered here; the sheets of effusive traps are distinguished by a greater thickness and a greater uniformity of composition. The hydrothermal occurrences within the upper horizons of the section of the lava are, in general, rather poor and monotonous, as compared to the lower parts of the section of lavas. Among the hydrothermal minerals, the most extensively developed here are zeolites (thomsonite and heulandite); less widespread are prehnite, quartz and chalcedony, and still less, calcite.

As has been shown above, the most promising from the standpoint of economically valuable accumulations of iceland spar, are the lower horizons of the lava series, while the probability of such findings in the upper horizons of lavas, composing the areas of divides between the Kochechuma, Kureika, Kotuya and Kheta rivers, is somewhat less. This does not mean, however, that only the marginal parts of the lava field within which mainly the lower parts of the section of the lava are exposed at the surface, are of interest as regards a possible occurrence of spar. It may be assumed that in the inner parts of the area occupied by the lavas, owing to the nature of the erosional section and the structure of the lava field, along with the upper horizons of lava, in the deep river valleys, in particular along the Kotuya and Kureika rivers, lower horizons of the section of lavas may be encountered, and possibly also the underlying tufaceous deposits.

As a result, these areas of the lava field may also be considered as promising with regard to the presence of spar.

The investigations carried out by the author within the northern parts of the Siberian platform show that the occurrences of spar are not restricted to the southern marginal part of the lava field. On the contrary, judging from the nature of the hydrothermal occurrences in those parts of the lava field which are confined to its eastern marginal part—to the basin of the Turu and Korvunchana, the left hand tributaries of the Kochechuma River, as well as to the middle course of the latter and thus situated 250—400 km from the Nizhnaya Tunguska River,—there are grounds to expect the discovery of accumulations of iceland spar of economic value.

This is confirmed by the considerable resemblance of the manifestations of minerogenesis in the lower and partly middle horizons of the lava series, exposed within the southern and eastern marginal parts of the lava field. In particular, in the basin of the Korvunchana River the same complex of minerals has been encountered which characterizes the southern marginal part of the lava field, at least in the portion of the valley of the Nizhnaya Tunguska River, adjoining the most promising (of those known at present) deposits of iceland spar. The principal, leading minerals of hydrotherms (quartz, chalcedony, calcite, zeolites) are in both cases represented by the same morphological varieties; and more than that, their ratios almost precisely repeat—not only in the most essential features but occasionally in details, too—what is at present known concerning the sequence of segregation of these minerals within the iceland spar deposits in the valley of the Nizhnaya Tunguska River, where the value of the latter is beyond any doubt.

Thus, in the course of itinerary survey no difference in principle has been revealed, in the composition of the complex of hydrothermal minerals characterizing the post-magmatic processes in lava sheets for a vast territory of the lava field adjoining its lower and eastern borders. In this way there are good reasons to expect here the discovery of iceland spar accumulations of economic value, similar to the deposits in the middle course of the Nizhnaya Tunguska River. Rather large areas of the lava field located north of the Nizhnaya Tunguska show a complete analogy to its more southerly portions from the standpoint of both structure and nature of manifestation of hydrothermal processes. In so far as it can be ascertained by

itinerary surveying, only the central parts of the lava field formed by the upper horizons of lavas possess a somewhat specific nature of hydrothermal occurrences and are therefore less promising from the standpoint of probable findings of economic iceland spar concentrations. The area of development of the upper horizons of the lava series, according to preliminary estimations, occupies about 50 000 sq. km; the possibility is not excluded that this figure has been overestimated, since the upper horizons of lavas have been encountered only within the highest water-divide elevations, the area of which is in general comparatively small.

The potentially spar-bearing part of the lava field thus constitutes about 270000—300000 sq. km. The lava field of the Siberian platform will certainly remain unequalled both with respect to the area occupied by lava sheets, and the regional distribution of spar occurrences. In particular, the world-renowned spar deposits of Iceland are associated with comparatively small lava fields, the area of which does not exceed 40 000 sq. km, a considerable part of this territory being covered with continental ice; the lava field of the Karroo formation, with which are also associated rather large-scale iceland spar accumulations, occupies an area of about 30 000 sq. km and in this respect is much inferior to the lava field of the Siberian platform.

Geological explorations of this province are quite imperative and the itinerary survey of the present author has been the first endeavour. The chief object of exploration should be to study in detail the section of the lava series as a whole and of individual areas of its development in particular; to determine the boundaries of the lava field and of the areal development of lavas. The problems of the stratigraphy and tectonics of the lava area must be developed and the specific features of mineralization as displayed in different horizons of the lava series, studied in detail, *etc.*

Such explorations will shed light upon the problems of the distribution of iceland spar in the various horizons of the lava series and thus help to find out the areas of the lava field most promising with regard to the possible occurrence of spar.

Received
12.X.1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

GEOLOGY

THE SINEVKA SALT DOME AND ITS GEOLOGICAL PLACE

By D. N. SOBOLEV and M. N. SHKABARA

(Communicated by V. A. Obruchev, Member of the Academy, 6. VII. 1944)

1. A new structure of the Romny type has recently been discovered in the central part of the Dnieper-Donetz depression in the Sinevka region of the Sumy district.

A detailed gravitational-variometric survey carried out in the Sinevka region has revealed a distinct local maximum of gravity located in the eastern part of a vast regional Romny minimum of gravity.

Its northern border passes through the regional centre—the Sinevka village, located 28 m from the town of Lebedin. In the west the line connecting the points with maximum gradients passes through the village of Podstavki; in the south-west, in the village of Birki, and in the east—near the Dovzhik village.

The Sinevka maximum is the shape of a compressed ellipse stretched from the south-west to the north-east approximately perpendicular to the strike of the long axis of the ellipse of the Romny and Dmitriev domes, 6.2×4.2 km in size. Like the Romny salt structure, in the Sinevka maximum an annular zone of large gradients can be distinguished, which encircles it like a ring. The maximum is distinctly delimited in the curvature map. By analogy with the Romny structure it may be assumed that the south-western part of the Sinevka maximum has the thickest cover of the cap-rock composed of rocks more compact than the surrounding ones, *i. e.* of fragments of diabase, clays, marls and, possibly, gypsum, diabase blocks of rather considerable size being possibly concentrated in this area.

To judge by the gradients and vectors of curvature in an eastward direction the surface of the supposed salt stock plunges 200—250 m, while the south-western part of the intrusion probably lies not deeper than 100 m; it is not possible, however, to make any conjectures as to the precise depths and thicknesses in this particular case, for lack of geological data, which makes any precise calculation impossible.

In the central part of the maximum, like in the Romny structure, the magnitude and direction of the vectors are subject to variation, which is probably accounted for also by the presence of diabase blocks.

It is of interest to note that the Sinevka maximum is the easternmost of the structures (Dmitriev, Romny) lying on the main longitudinal dome line of the North-Ukrainian basin and at the point of intersection of the latter with the transverse fracture line, passing from Kremenchug through the Bogach dome, the Sinevka maximum and, probably, to Shtepovka.

The data of gravitational-variometric survey concerning the Sinevka structure, supported by data of seismic work, make it possible to carry on shallow drilling with the purpose of quantitative calculations.

The discovery of the Sinevka structure goes once more to confirm that the Dnieper-Donetz depression is a more complicated structure, and makes us believe

that in the course of further exploration some new structures will be discovered in the Romny—Sinevka region, as well as further eastward, which like the Sinevka structure will be both of geological and practical interest.

2. The Sinevka salt dome is located in the North-Ukrainian basin constituting the western part of the Dnieper-Donetz trough which is a post-orogenic near-mountainous depression of the Donetz ridge. The reduction in size and disappearance of the Amidocean geosynclinal basin began after the Asturian phase of Hercynian orogenesis which produced the Hercynian core of the ridge. Outside the core, partly beginning with the close of Middle Carboniferous, started the deposition of variegated molasses, which in the form of more continuous series filled the Dnieper-Donetz trough in the Upper Carboniferous and possibly at the beginning of the Lower Permian. In the eastern part of the trough a number of isolated salt lakes were already formed at that time, which deposited gypsum-salt stocks amidst a field of variegated rocks. On the north-western posthumous border of the Donetz ridge the anhydrite-salt formation began somewhat later, during the Lower Permian.

These lakes—sink-holes—seem to have been located along two fracture lines, passing along the Dnieper-Donetz trough, one along its axis, another along the line separating the depression part of the trough from its southern (Kiev—Poltava) shelf. The later tectogenic movements of the late Hercynian (Zaal), Cymnerian, sub-Hercynian, Laramie and Pyrenean phases transformed these fractures into contrary overthrusts, which in the northern dislocation were directed southwards, and in the southern, northwards. In connexion with the contraction of the southern zone of the trough depression, which produced these overthrusts, along the axis of this zone a median, arch-like uplift was formed, accompanied by a longitudinal fracture and an effusion of heavy magmas, that brought about the formation of positive gravity anomalies. Similar effusions occurred also along the two overthrust lines. The same compression transformed the salt stocks mentioned into salt domes: the bulging out of these domes was accompanied by the formation of tectonic breccias enveloping the domes; in the composition of these breccias an important part belongs to fragments and blocks of diabases.

Along with this, in front of the zones of dome expressions, south of the northern and north of the southern one, compensation downwarpings were formed, filled with thick sediments, those of a Palaeogene age inclusively. North of the northern and south of the southern overthrust line, as a result of the same transverse compression, local dome elevations of rocks were formed. Thus the interoverthrust cataclase with an axial arch and local dome warpings of the Palaeozoic and with marginal fore-dome depressions is bounded both in the north and in the south by a double row of domes—chiefly along the overthrust line and an additional row from the external side of the latter.

In the North-Ukrainian basin the dome structures are concentrated mainly in the southern zone of its middle portion, where they fringe from all sides the Lovitz bath. In the south the main dome line passes from the Poltava uplift through the Bogach, Mirgorod (?), Romodan, Isachki, Pozdniakov, Priluki (?) domes; the additional dome line passes through the Reshetilov (further south-westward there are some more uplifts near the mouth of the Goltva and near Ostapye), Lubny, Yaziny domes.

In the west the bath is bounded by the Ichnya-Bakhmach line, where buried structures are supposed to exist (neither the grounds for this assumption nor the type of the structures are known to the author). In the north the main line consists of the Dmitriev, Talalaev, Romny, Sinevka, Parkhomov domes; the additional line, of uplits at Khmelevo, Shtepovka and Akhtyrka. The eastern closure of the bath follows the direction from the Parkhomov dome through the structures of Vodyanaya and Kochubeevskaya, lying on the line of the axial arch of the bath, on which inside it an uplift of the Carboniferous has been established at Glinsk.

The domes were formed at the points of intersection of the longitudinal and the transverse dislocations. Lying on the Ishachki—Romny transverse line are: an

uplift of the Palaeozoic at Lubny, the Ishachki dome, the southern fore-dome depression, the palaeozoic uplift in the middle of the bath at Glinsk, the northern fore-dome depression, the Romny dome, the uplift of the Palaeozoic at Khmelevo. Until now this line has been considered as the main transverse line of oil-bearing structures.

The main transverse tectonic line is the one marking the zone of Saksagonides, traced from Krivoy Rog through Kremenchug to the mouth of the Goltva, where it plunges under the depression of the North-Ukrainian basin, on the other side of which the same direction is followed by the northern end of Saksagonides of the south-western belt of Central Russian magnetic anomalies. In the Lokhvitz bath and in its vicinity on this line there are located: the Ust-Goltva structure, Ostapye structure, Bogach dome, Rashovka-Sosnovka maximum of gravity, Sinevka dome, stretched in the direction of this transverse dislocation, the uplift at Shtepovka, the belts of magnetic anomalies on the Seim, south and north of Rylsk, west of Lgov.

The Sinevka dome lies at the intersection of the main longitudinal dome line of the North-Ukrainian basin with the main transverse line, on the north-eastern border of the Lokhvitz bath. Its geological structure is probably analogous to that of the Romny dome, to which it is likely to compare with regard to the possible occurrence of oil.

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Received
6.VII. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MINERALOGY

ON SOME HIGH-ALUMINA MINERALS IN A BLOCK OF ELECTROCORUNDUM

By N. E. FILONENKO

(Communicated by D. S. Beliankin, Member of the Academy, 25. IX. 1944)

The artificial abrasive material—electrocorundum—is a technical rock consisting of corundum and of minerals associated to it: it is produced by a reduction smelting of bauxite in an arc electric furnace. A petrographical investigation of domestic electrocorundum made by the author in 1936 demonstrated that, according to chemical composition and conditions of formation and crystallization, electrocorundum fits within the conditions of equilibrium of the ternary diagram, $CaO—Al_2O_3—SiO_2$ (1). The electrocorundum produced by the «Ilyich» works at Leningrad, smelted from Tikhvin bauxites rich in SiO_2, consists essentially of corundum and mullite; while the electrocorundum of the Chelyabinsk abrasive works which make use of bauxites of the «Krasnaya Shapochka» deposit, consists of corundum and anorthite. The formation of anorthite is evidence of a rather low SiO_2 content in the original bauxite; it is desirable from the technological standpoint, since the formation of anorthite consumes one third of the amount of Al_2O_3 required for the formation of mullite. As a result, the electrocorundum of the Chelyabinsk abrasive works, though having the same Al_2O_3 content, compares favourably with the corundum of the «Ilyich» works in its high corundum content and higher abrasive capacity.

During the last three years the Chelyabinsk works organized a smelting of low-grade bauxites with a SiO_2 content up to 10 per cent and a CaO content of 1.0—1.2 per cent, which naturally resulted in a lowering of the grade of the product. In attempting to improve the quality of the electrocorundum, the works introduced into the charge an enriching material (electrocorundum dust with 87 per cent Al_2O_3), thereby raising from 87 to 92 per cent the Al_2O_3 content in pole samples of the melt selected in the process of smelting for controlling the grade of the product obtained. This measure, however, failed to bring about the favourable effect it was expected to produce and did not increase the abrasive capacity of electrocorundum. In order to make out the cause of the low abrasive capacity of electrocorundum the author undertook in 1943 a petrographic study of electrocorundum blocks of an «improved grade» (with a high Al_2O_3 content).

Samples taken from different places of the block were subjected to microscopic analysis in thin sections. The examination yielded the following results: the electrocorundum in the middle of the block was composed of pink corundum and coloured lamellae with a sharp pleochroism: green lamellae—with ε-green, ω-greenish-grey, and blue lamellae, with ε-blue, ω-violet-grey. The lamellae were arranged in a fringe around the crystals of corundum or formed plank-like aggregates at the junctions of the latter; the spaces between the lamellae were filled up with anorthite.

In the lower part of the block the lamellae were pale-blue in colour, the colour being concentrated at the edges of the lamellae and, finally, at the contact with the carbon hearth of the oven, they were quite colourless. In these varieties anorthite did not segregate, and instead of it yellow grains of calcium carbide were observed between the plank-like aggregates.

Along with this a gradual absorption of corundum with plank-like aggregates was noted upon a shifting from the centre to the bottom of the block. In the centre of the block the corundum crystals had diffuse outlines due to corrosion of the melt with a subsequent formation of lamellar aggregates; towards the bottom of the block the plank-like aggregates showed a tendency to predominate over the corundum which was here the shape of rounded grains. Upon approaching the bottom, the process of solution of corundum would become more intense, and the remains of its crystals were seen as grains of irregular shape within the body of lamellar aggregates.

Finally, the bottom of the block, at the contact with the bottom, was found to be composed of large plank-like aggregates, crossing at random; polygonal areas between these were filled with small tabular colourless crystals of corundum of a secondary generation.

The green, blue and colourless lamellae which had been separated out by the author were subjected to chemical analysis, their refractive indices, density and abrasive capacity were determined. The results of the analyses and tests are tabulated in tables 1 and 2.

Table 1

Chemical Analysis of Crystals in Weight per cent

	Lamellar mineral			Corundum
	green	blue	colourless	
Al_2O_3	80.22	84.02	91.04	97.59
SiO_2	2.08	1.98	0.36	0.34
TiO_2	8.78	4.48	0.513	0.58
Fe_2O_3	0.75	0.86	0.86	1.78
CaO	8.67	8.71	7.32	0.02
Total	100.50	100.05	100.11	100.31

Table 2

Optical Properties, Density and Abrasive Capacity of the Crystals Separated out

	Colour	Habit	N_o	N_l	$N_o - N_l$	Density, g/ml	Abrasive capacity, g
Lamellar mineral	green	basal lamellae	1.790	1.780	0.010	3.64	0.117
»	blue	do	1.785	1.776	0.009	3.61	0.109
»	colourless	do	1.757	1.750	0.007	3.54	0.092
Corundum	pink colourless	rhombohedra, tables, grains	1.768	1.760	0.008	3.96	0.156

It may be seen that in the light of N.A. Toropov's data and according to their chemical composition, the lamellae should be referred to a high-alumina calcium aluminate—$CaO \cdot 6Al_2O_3$ [2]. The colour of the lamellae is due to the presence of titanium oxide, Ti_2O_3 (as it is known, the solution of salts of trivalent titanium have a blue-violet colour), which seems partly to replace alumina in the aluminate:

$$\text{green lamellae} — CaO \cdot 5.6Al_2O_3 \cdot 0.4Ti_2O_3$$
$$\text{blue lamellae} — CaO \cdot 5.8Al_2O_3 \cdot 0.2Ti_2O_3$$

It is noteworthy that the pink colour of the corundum depends also on the presence of titanium dissolved in it in a very insignificant amount, apparently in the form of titanium dioxide, TiO_2 (as well-known, the salts of a titanic acid are pinkish-yellow in colour). This phenomenon of simultaneous staining by titanium of common corundum and its lamellar variety, as the author then interpreted $CaO \cdot 6Al_2O_3$, was observed in 1937 in the course of microscopic examination of electrocorundum produced by smelting of alumina with addition of TiO_2 and CaO in a laboratory electric arc furnace. The addition of TiO_2 alone invariably produced a pink colour in corundum, its excess was reduced and observed among corundum crystals in the form of black veinlets; upon introducing CaO, around the pink crystals of corundum blue-green lamellae were formed, which the author supposed to be a special variety of corundum [3].

From its optical properties the lamellae closely approach corundum being sharply different from $CaO \cdot 6Al_2O_3$ synthesized by N. A. Toropov by treating $NaO \cdot 11Al_2O_3$ with melted calcium chloride [2].

The density of the lamellae is higher than that of the aluminate synthesized by N. A. Toropov [2].

From what has been said above it may be assumed that in the given case we have to do to a new modification of $CaO \cdot 6Al_2O_3$ or crystalline alumina*.

The abrasive capacity of high-alumina calcium aluminate is 30—40 per cent lower than the abrasive capacity of corundum; its formation consumes moreover six times the quantity of Al_2O_3 consumed in the formation of anorthite, which facts account for the lower abrasive capacity of the «high grade» electrocorundum.

Aside from a more precise determination of the composition, optical properties and conditions of formation of high-alumina calcium aluminate in an electrocorundum block, the study here reported permits of the following conclusions:

1. Under conditions of reduction smelting and deficiency of SiO_2 for binding the total CaO to anorthite, the crystallization of the high-alumina melt assumes the following course: first, corundum separates out from the melt; then its crystals are subjected to dissolution under formation of $CaO \cdot 6Al_2O_3$. In the middle of the block the solution of corundum does not proceed to the end, and the residual melt solidifies as anorthite. Thus the ultimate products of crystallization of the groundmass of electrocorundum will be: corundum, $CaO \cdot 6Al_2O_3$ and anorthite. In the lower parts of the block where the reducing agent is present at a larger amount, SiO_2 is completely reduced to metal, the solution of corundum involves deeper layers of the block and as a result the electrocorundum making up its lower portion is found to consist of $CaO \cdot 6Al_2O_3$, in the lamellae of which the remains of corroded corundum crystals are noticeable. At the contact with hearth, where the reducing agent is present in excess, the process of solution of corundum is completed and $CaO \cdot 6Al_2O_3$ crystallizes as large well-developed hexagonal lamellae, in which it is impossible to trace the previously segregated corundum crystals; unlike this, the latter is segregated as small tabular crystals of the secondary generation, filling up the spaces between the lamellae of $CaO \cdot 6Al_2O_3$. In the varieties rich in CaO no corundum at all is present, and the excess of CaO over that necessary for the formation of $CaO \cdot 6Al_2O_3$ is reduced to carbide, which is the cause of the disintegration of the varieties of electrocorundum at the contact with the hearth of the furnace.

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* Cf. in this connexion the data of D. S. Beliankin and V. V. Lapin on Zestafon corundum, with its low N_0, varying from 1.764 to 1.746, and with a content of over 5 per cent CaO [3].

In this way under the conditions of reduction smelting and a deficiency of SiO_2 necessary to bind the total CaO to anorthite, the crystallization of the aluminous melt does not proceed in accordance with the conditions of equilibrium in the ternary system $CaO—Al_2O_3—SiO_2$. A new mineral phase, the melting of which is accompanied with decomposition is thus formed; it is a high-alumina calcium aluminate, whose range of stability seems to approach those of corundum and anorthite.

2. At the close of the reduction process, when the relative TiO_2 content increases as the SiO_2 content in the reaction medium decreases, the reduction of the two oxides proceeds simultaneously. This phenomenon which is of considerble technical importance is easily observable under the microscope, which should be applied to express-analyses of pole samples taken from the melt with the object of controlling the process of reduction.

In conclusion it should be recommended in all cases of industrial production to carry out determinations of the CaO content in the bauxite, in order that the technologists in proportioning the reducing agent be able to evaluate the amount of SiO_2 to be left in the melt for binding CaO to anorthite, and thus to improve the quality of electrocorundum.

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Received
25. IX. 1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

PALAEONTOLOGY

PRIMARY FACTORS IN THE DEVELOPMENT OF THE CORALS *ZOANTHARIA*

By N. N. YAKOVLEV, Corresponding Member of the Academy

The factor predominant in determining the morphogenesis of the Palaeozoic *Tetracoralla* is that their habitat lies within the region of shallow sea-waters subject to the action of the surf (for instance, the fastening of the Silurian coral *Alleynia* to the crests of ripple marks on the limestone in the Pre-Baltic region). The conic polyps of the Silurian period often attached themselves by means of root-like processes diverging at different levels on all sides of the polyp body. Being no sufficiently solid one, this mode of fastening was replaced by way of natural selection, by another, in which the lateral surface of the polyp was directly fastened to the substratum. In order to prevent sea bottom mud from penetrating into the inner cavity of the polyp body during its further growth, the polyp turned the aperture of the cup upwards, away from the substratum, thus assuming the curved shape of a horn (Fig. 1). The primary septa (*S*), originating from the lower tapering end of the polyp acquired a curved shape together with the latter. The aperture of the polyp cup is mostly situated at a right angle to its concave surface (or forms a larger angle with it than with its convex surface). The secondary septa (*aa*) which develop after the primary ones and begin in the polyparium at a higher level than the latter, are perpendicular to its aperture, or nearly so. In the two opposite quadrants, *i. e.* in two (of the four primary) chambers adjacent to the concave surface, the secondary septa may run perpendicular to the aperture, since the line of the concave surface coinciding with the trace of the primary, «counter» septum here located, is perpendicular to the aperture. Here the traces of the secondary septa run parallel to the trace of the primary counter septum, and as they develop within triangular species, owing to the polyp being cone-shaped, the secondary septa are at the same time necessarily at an angle and run pinnately to the alar primary septum, whih bounds the triangle of the quadrant on the lateral surface of the polyp.

In the main quadrants adjacent to the convex surface, the secondary septa can no longer run perpendicular* to the aperture, since here the primary alar septum serving as guide to the secondary septa is not perpendicular to the aperture: the secondary septa develop deviating as little as possible from a direction perpendicular to that of the aperture, *i. e.* they run parallel to the alar septa and at the same time are necessarily at an angle, *i. e.* run pinnately to the trace of the main primary septum situated on the convex surface. Thus, a characteristic feature of the *Tetracoralla*, namely a primary and parallel arrangement of the secondary septa with respect to the primary ones (Kunt's law) as well as their origination in pairs,

* This direction is indicated in Fig. 1 by a dotted line.

is due to a topographic correlation. The formation of septa is accompanied by the origination of so-called fossulae, ditches (*F*, Fig. 2). The latter are formed on the alar and the main primary septa at the site of the origination of new secondary septa: there are four such points which are located on one side of the alar primary septa and on both sides of the main primary septum. This is due to the fact that at these points there are spaces which appear triangular in shape when viewed laterally, gradually widen as the coral grows upwards, and in which four new septa are found to originate when these spaces have reached the necessary width. Owing to the sinking of the soft body of the polyp into these triangular spaces, the end of the septum which is nearest the centre deviates under the pressure of the soft tissues from its radial direction and joins the septum which was formed previously.

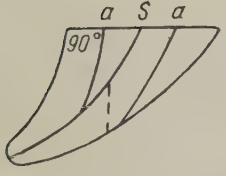

Fig. 1

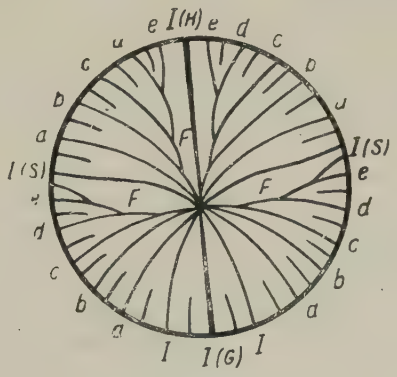

Fig. 2

This results in the formation of a chamber between the primary and its nearest secondary septum, this chamber exceeding in width the interseptal chambers formed between the adjacent secondary septa. These chambers formed with the primary septum are precisely the fossulae mentioned above, and the sum of the two fossulae formed with the main primary septum makes up one main fossula which is larger and more markedly manifest than the two single alar fossulae. In case of transformation of the initially conic polyp, the prototheca, upwards in a cylindric polyp with a uniform diameter, the septa cease increasing in number, develop an equal growth and show no longer bilateral symmetry in their arrangement. Thus, in this case the second part of the specimen's life is characterized by a radial arrangement of the septa (sometimes inadequately termed «pseudoradial»), which is, however, different from the radial arrangement in the more recent *Hexacoralla*. In the latter this arrangement consists of several cycles, thus differing from that of the *Tetracoralla*, in which the septa are arranged in one cycle (except for the small septa which develop simultaneously between the larger septa at the end of the growth of the individual).

In the more ancient *Tetracoralla* the polyps are often the shape of a wide cone, which is a disadvantage, for according to the law of mechanics a cone is the more readily overturned, the larger its angle. The excessive widening of the polyp is compensated for by the so-called process of rejuvenescence, in which the prototheca dies away having gemmated inside the cup of the mother polyp without separating from the latter. If the process is repeated several times it results in the formation of a bead-like polyparium; the old term «rejuvenescence» has met some objections and, indeed, it might be better to call the process «periodic reduction». The horn-like curved polyps have an orientated distribution on the bottom of the sea, as has been proved by their previously mentioned occurrence on the crests of ripple-marks, with their convex surface directed towards the surf. This is the most advantageous position from the viewpoint of the polyp's mechanical stability, for owing to it the polyp becomes the least subject to the force of the breaking waves. A similar orientation is apparently acquired through the action of rheotropism and possibly also through natural selection.

More recent, Carboniferous polyps sometimes acquire another adaptational property, namely, the transverse section of the polyp no longer remains circular, but becomes elliptical in shape, so that the long axis of its section runs along the symmetry plane of the polyp, and, consequently, in the direction of the surf. In this way the polyp is better protected against the force of the waves and acquires a greater stability. By the end of the Palaeozoic, owing to intense orogenesis, large quantities of terrigenous clastic material were thrust into the sea, which affected most unfavourably the existence of the *Tetracoralla*. Some of their groups died out, others migrated to greater depths and to calm waters; polyps having the shape of wide cones were superseded by such as formed narrow cones or were cylindrical in shape, and developed the capacity of fastening themselves with their wider base, unlike the *Tetracoralla*, which are the shape of narrow cones. The last to be explained is the radial arrangement of the septa characteristic of the *Hexacoralla* and different from the bilateral symmetrical arrangement observed in the *Tetracoralla*. But the radial symmetry is a usual phenomena in fixed animals and may even be regarded as being peculiar to them, for it is found to occur not only in *Coelenterata*, but also in *Echinodermata* (crinoids, *etc.*) and even, though rarely, in molluscs (rudists), brachiopods (*Richthofenia*), and as a matter of course, in fungi and fungus-like calcareous algae, the *Acetabularia*, *etc.* Mushrooms, in particular, have radial laminae which support the mushroom flap from below and which are arranged in cycles in the same way as the septa in the *Hexacoralla*. This cyclic arrangement is natural in a body having a circular transverse section which increases in diameter with the growth of the individual, with the result that the sectors between the septa increase in width at the periphery, thus making the introduction of new septa not merely natural but indispensable.

The septa in the *Hexacoralla*, at the time of their origination from the *Tetracoralla*, have one peculiarity which needs to be explained. To judge by the illustration offered by Weissermel, by certain Permian corals (precisely those which are supposed to have given rise to the *Hexacoralla—Gerthia*, *Plerophyllum*, *etc.*), this peculiarity consists in the formation of a peculiar club-like thickening at the distal edge of the septa, clearly visible in cross-sections of the polyps. This thickening is observed neither in the more ancient *Tetracoralla*, nor in the more recent *Hexacoralla*. Apparently this thickening is due to a rapid transition of the *Tetracoralla* to the *Hexacoralla*, which causes a certain shortening of the septa resulting from a precocious interruption of their radial growth, owing to the shape of the polyp being changed from that of a relatively wide cone to a narrow cone or a cylinder. At the same time the growth hormone which produced the radial growth of the septa to a definite length continues being secreted in the same quantities as before, either owing to heredity or inertia, and results in an excessive deposition of calcareous substance at the distal edge of the septa. This excess causes the above-mentioned club-like thickening. In the course of time the action of the hormone and the deposition of calcareous substance once more regain their balance, causing the club-like thickening to disappear.

Thus, the factors determining the morphogenesis of the *Zoantharia* corals are as follows: the effect produced by the arrangement of previously formed organs on those developing later (topographic correlation), tropisms, natural selection, hormones, and in the *Tetracoralla*, primarily the life in shallow water subject to the action of the surf.

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Received
25. VI. 1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

MICROBIOLOGY

SENSITIVITY OF *PARAMAECIUM CAUDATUM* TO TOXIC AGENTS AT VARIOUS STAGES OF ONTOGENESIS

By A. KOVALENOK

(Communicated by A. A. Zavarzin, Member of the Academy, 9. XII. 1944)

A number of special studies ([1–4]) have been devoted to the problem of cell ontogenesis, in which B. Tokin's conceptions were supported and developed in more detail.

By the ontogenesis of the cell we are to understand the processes of its development since the moment of origination as a result of division till the moment it gives rise to two daughter-cells. At the period between the two divisions a series of biological changes take place in the cell, which bring about division again. The so-called resting stage of the cell is, as a matter of fact, a stage of its maximum activity. It is at the time interval between two consecutive divisions that the cell undergoes a number of complicated biochemical, morphological and physiological transformations which finally lead to a new division.

Consequently, at the different moments of its ontogenesis the cell is found to vary both in its morphological and physiological characters. It is but natural to anticipate therefore that it will respond in a different way to a similar dose of a chemical or physical harmful agent. This reasoning is applicable both to the cells of metazoa and to unicellular organisms.

G. Gorbunova investigated the effect of quinine upon *Paramaecium caudatum* at various stages of the latter's ontogenesis. The experiments consisted of two sets, the quinine solutions being applied at 1 : 18000 and 1 : 35000 concentrations, respectively.

One lot of infusoria were taken under experiment 2 and 19 hours after division, and another 6 and 12 hours after division. From the records made was calculated the mean sensitivity of the infusoria under experiment.

The sensitivity of the infusoria of the two lots to quinine, applied at 1 : 18000 and 1 : 35000 concentrations, was found to differ significantly. Thus, the data obtained by Gorbunova should be interpreted to mean that the sensitivity of *Paramaecium caudatum* to quinine changes within the time interval between two divisions.

The present author studied the effect of onion phytoncides ([5]) upon *Paramaecium caudatum* at the various stages of its ontogenesis.

The experimental clone was obtained from a single individual; as nutrient medium use was made of hay infusum. The culture was kept at a room temperature ranging about 18° C. The experiments were carried on at 18° C.

The agent tested was a solution of crystalline phytoncide obtained from *Alium cepa* bulbs, which had kindly been put at our disposal by I. Toropzev. The concentration of the solution tested was 1 : 200 000.

The rate of division of the infusoria was established previously to the experiment. In *Paramaecium caudatum* of the clone bred at the laboratory the time interval between two consecutive divisions was found to be equal to 17 hours. For the experiments we picked out from the culture of *P. caudatum* such individuals as were at the stage of division; each dividing individual was set apart in a drop of hay infusum placed up on a watch-glass. The moment of a complete fission of the infusoria into two sister individuals was precisely recorded. Then, after a definite time interval, which varied according to the object of the given experiment, the infusoria were individually transferred to an object glass in as small a drop of hay infusum as possible, and under visual control, to the drop of the infusum was added a drop of the toxic substance; this moment was recorded by setting into operation the stop-watch. The stoppage of the lashings of the ciliae served as a criterion of the lethal effect of the solution tested. The time interval between the application of the toxic substance and the death of the infusoria was recorded as a criterion of their sensitivity. All experimental sets were run under identical condition, the dose of the poison tested being also identical in all cases. The results of the experiments are summarized in the table below.

Time after division, in hours	Number of experiments	Variation range	$M \pm m$ (in sec.)
1	49	16—44	29.65 ± 1.07
2	52	20—55	37.30 ± 0.26
6	49	20—55	37.05 ± 1.3
8	49	22—70	38.55 ± 1.4
11	49	21—63	42.5 ± 1.35
17	52	28—97	49.41 ± 0.15

From the figures obtained in all experimental sets, the mean sensitivity was calculated for infusoria taken under experiment at different stages of ontogenesis (see table). In the first set was investigated the sensitivity of infusoria taken under experiment 2, resp. 17 hours upon division. The mean $M_2 = 37.30 \pm 0.26$ sec.; $M_{17} = 49.41 \pm 0.15$ sec. It is quite evident that the difference between the sensitivity of the infusoria investigated 2 hours upon division and of those tried 17 hours upon division is significant, for it exceeds its possible error 8.07 times.

Having established a significant difference in the sensitivity of the infusoria at two development stages, we undertook to find out the variation in sensitivity at earlier stage of ontogenesis and at the intermediate stages between that of 2 and that of 17 hours following division.

With this aim in view we arranged two experimental sets, one of them involving infusoria taken 8 hours upon division, while the other sister individual was put to test 11 hours after division.

As will be seen from the same table, the mean sensitivity of the infusoria was found to be $M_8 = 38.55 \pm 1.4$ sec. and $M_{11} = 42.50 \pm 1.35$ sec., respectively.

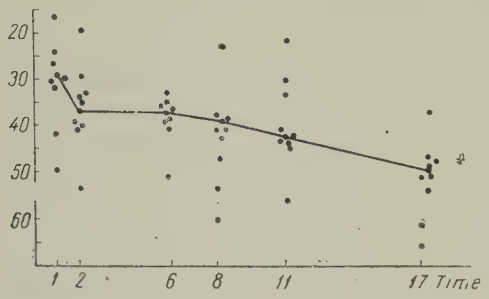
In this case the difference between M_8 and M_{11} is no sufficiently significant one, for it is only 2.07 times its possible error. This should probably be put down to the fact that within the time interval between 8 and 11 hours following division the organism of the infusorium undergoes but very slight changes which defy the recording method here applied. Quite different is the picture offered by the third set of experiments, where the response of the infusoria to the toxic substance was tested 1 hour and 6 hours upon division. Here the mean $M_1 = 29.65 \pm 1.07$ sec.; the mean under comparison, $M_6 = 37.30 \pm 0.26$ sec. We see that the difference is fully significant this time, for it is found to exceed its error 6.8 times.

Thus, we have obtained a biometrically significant difference between the sensitivity to onion phytoncides in infusoria tested 2 and 17 hours after division.

From the graphical interpretation of the above data plotted in the figure it may clearly be seen that individuals of *Paramaecium caudatum* do not respond in

the same way to onion phytoncides when tested at different stages of the ontogenesis, and that their sensitivity is subject to variation between two consecutive divisions, decreasing at the approach of the next division, and increasing again, after the division was complete.

According to the data obtained by Gorbunova, the sensitivity of *P. caudatum* to quinine changes within the interval between the divisions. However, the sen-



Variation in sensitivity of *Paramaecium caudatum* to onion phytoncides

sitivity to quinine was not found to decrease in a proportion inverse to time, but unlike this, to increase and decrease more than once during the period between the two divisions.

In spite of a slight discrepancy in the shape of the curves, our data are in good agreement with the data obtained by Gorbunova with *P. caudatum* and frog eggs at the stage of fission: they demonstrate that the sensitivity of cells to the effect of chemical-physical agents is subject to variation during the time interval between two consecutive divisions. The sensitivity of *P. caudatum* to the phytoncides of *Alium cepa* decreases at the approach of a new division, and increases again as soon as the division is complete.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

PLANT PHYSIOLOGY

GROWTH OF ROOTS AS STIMULATED BY CERTAIN ORGANIC ACIDS

By A. V. BLAGOVESHCHENSKY and A. J. KOLOGRIVOVA

(Communicated by A. A. Richter, Member of the Academy, 23.II.1945)

Magitot ([2]) obtained successful grafts of the cornea taken from the eye of a live human subject and preserved during 8 days at a temperature of several degrees above the freezing point. On the basis of these experiments, V. P. Filatov ([1]) made in 1934 his first attempt to utilize for the same purpose autoptic material kept for some 1—8 days at a temperature of 2—4°C. The material proved favourably to compare with the live one: the number of transparent successful grafts increased more than twice. Along with a better rate of healing, another phenomenon was also observed, which, though known before, appeared especially striking when the cornea was subjected to cold prior to transplantation, namely, the clearing of the wall-eye around the transplant. This phenomenon was laid by Filatov ([3]) at the base of his new method for healing not only eye diseases, but also of other tissues or organs.

Of special importance is the observation made by Filatov, *viz.* that the healing action is produced not only by the transplantation of chilled tissues, but by the injection of extracts obtained from them. The kind of tissue applied was found to be indifferent: preserved leaves of aloe produced the same therapeutical effect as, for instance, preserved placenta.

According to the working hypothesis developed by V. P. Filatov, the tissue preserved at low temperature seemed to give off into the recipient organism certain substances which stimulate resorption and regeneration processes, and cause such processes of reconstruction as may lead to recovery under pathological conditions.

These substances were termed by V. B. Filatov «factors of conservation», or «biogenic stimulants»; in his studies he more than once emphasized the fact that the processes of their formation are by no means identical with those of *post-mortem*, autolysis of tissues or cells.

Our studies which were conducted in the closest cooperation with V. P. Filatov, have shown first of all that biogenic stimulants are neither proteins nor enzymes. We have arrived at this conclusion by studying extracts of germinated seeds of *Phaseolus aureus* Raxb. which had developed rootlets 0.5—1.0 cm long and were chilled during 10 days at a temperature of $+2^{\circ}$. When taken out from the refrigerator the seedlings looked quite fresh and healthy. They were ground, the juice filtered off and the proteins coagulated by boiling. The filtrate was sterilized in an autoclave at 120°C and found to contain neither proteins nor enzymes. The therapeutic effect of this extract was tested by Dr. V. V. Skorodinsky on Pendina ulcer and inflammations of the cornea of herpetic type, and yielded fully positive results. An identical therapeutic effect was obtained with extracts from which, besides proteins, were also removed all the substances which are precipitated by phosphotungstic acid in acid medium.

It could be deduced from this that the biogenic stimulants are of the nature of organic acids. Their formation could be traced in the following way. By chilling the tissues of an organism are made to pass from their normal conditions of exchange, where the rates of individual biochemical reactions are strictly timed, as a result of the whole phylogenetic development of the organism. As the temperature is made to sink, the thermal coefficients of the hydrolytic and synthetic reactions grow faster than the thermal coefficients of the reactions of oxidation and reduction. Accordingly, the substances are formed which do not occur under normal conditions and, like any substance which is foreign to the organism, they have a stimulating effect upon the protoplasm when present at a definite concentration, or an inhibiting or even toxic effect when present at another concentration. It is easy to imagine that aspartic acid, for instance, which is consumed by the organism under normal thermal conditions in the synthesis of proteins, would under the influence of cold undergo oxidizing desamination and be converted into malic acid and the latter would give in the processes of dehydration and reduction fumaric and succinic acids.

In a similar way, phenyl-alanin may produce cinnamic acid, tyrosin, paracumaric acid, *etc.*

Our aim was to find out whether such compounds as aspartic, succinic and cinnamic acid will produce an effect similar to that of V. P. Filatov's biogenic stimulants. As a suitable experimental material, use was made of rootlets of the germinating seeds of *Phaseolus aureus*, which had preliminarily been tested for the effect of biogenic stimulants and gave clear positive results; the increment of the rootlets was recorded after 24 and 48 hours. The measurements were made with the aid of a beam-compasses to 0.1 mm. The seeds were germinated in Petri dishes, directly in the solutions to be tested, or in distilled water, after 24-hour-long swelling in such solutions and washing in water. The experiments were made at room temperature (16—18°) and in a thermostat (30°). In some instances, instead of measuring the linear increment, we determined the increase in the volume of the seedlings by measuring in ordinary burettes the volume of water displaced by them. The two methods gave fully concordant results which showed that the acids tested had in fact stimulated growth, like the biogenic stimulants do. Let us present the results of some of our experiments.

In the experiment of 19. IX. 1944 there were taken 20 seeds of *Phaseolus aureus*, which were placed into Petri dishes; 10 cm^3 of 0.001 *M* solution of the acids were poured into each and put into a thermostat at 30°. Measurements of the length of the roots, made after 24 and 48 hours, gave the following results (in mm, Table 1).

Table 1

Period, in hrs.	Water (control)	Succinic acid	Fumaric acid	Oxalic acid	Aspartic acid
24	11.4 ± 0.4	15.2 ± 1.0	5.8 ± 0.2	0	5.6 ± 0.2
48	21.4 ± 0.7	26.5 ± 1.9	—	0	11.5 ± 0.6

The stimulating effect of succinic acid is pronounced here with a sufficient clarity; oxalic acid has completely suppressed the development, while fumaric and aspartic acid at the given concentration caused a slowing down of the growth of the root. Measurements of the change in volume have shown that the increment under the influence of aspartic acid was somewhat greater than in the control, and weaker concentration was connected with a higher stimulating effect. The derivative of phenyl-alanin—cinnamic acid, gave still more interesting results, for at concentrations from *M*/1000 to *M*/4000 it was found to inhibit the increment in the volume of the roots, and at concentrations

from *M*/6000 to *M*/8000, it proved a stimulant. The experiment was carried on at 30°C, the seedlings being all the time immersed in the solution tested (Table 2).

Table 2

Stimulating Effect of Aspartic and Cinnamic Acids

Stimulant	Concentration	Volume of 20 seedlings, in cm^3			
		after 24 hrs.	after 48 hrs.	increment during the second 24 hrs.	increment as compared with control
Water (control)	—	3.6	4.6	1.0	0
Aspartic acid	*M*/1000	3.6	4.8	1.2	+0.2
	M/2000	3.2	4.6	1.4	+0.4
	M/6000	2.6	4.7	2.1	+1.1
Cinnamic acid	M/1000	3.8	4.2	0.4	—0.6
	M/2000	3.2	4 0	0.8	—0.2
	M/4000	4.2	5.1	0.9	—0.1
	M/6000	3.3	4.4	1.1	+0.1
	M/8000	3.0	4.3	1.3	+0.3

The effect of concentration was studied in some detail in the case of succinic acid. In the experiment of 23.X the seeds of *P. aureus* were made to swell in water and in the solutions of succinic acid of different concentration. On October, 24 the seedlings were rinsed in distilled water and left in water at 16°C. Measurements were made after 48 and 72 hours. The results were as follows (Table 3).

Table 3

Stimulant	Concentration	Length of roots, in mm	
		47 hrs.	72 hrs.
Water (control)	—	9.1 ± 0.9	14.0 ± 1.8
Succinic acid	*M*/3000	10.7 ± 1.0	18.6 ± 2.0
	M/4000	12.4 ± 1.2	$19,2 \pm 2.1$
	M/5000	9.9 ± 1.0	17.2 ± 2.0
	M/6000	9.0 ± 1.0	12.1 ± 1.6
	M/8000	10.1 ± 1.2	17.5 ± 2.6

In another experiment in which the seedlings, after soaking in the acid, were kept at 30°C in distilled water in Petri dishes, the following length of the roots was recorded (mm): water 23.4 ± 1.2; *M*/3000 26.8 ± 1.2 mm; *M*/15000 29.6 ± 1.4 mm. The effect of temperature is a very sharp one. Thus, for example, in the experiment of 31.X the seeds of *P. aureus* have swollen in water and in the solutions of succinic acid *M*/3000 and *M*/5000. On November 1st they were rinsed and kept on developing further in water in Petri dishes at 15—16° and at 28—30°. The results are summarized in Table 4 (mm).

Table 4

Stimulant	Concentration	28—30°				15—16°			
		24 hrs.		48 hrs.		72 hrs.		96 hrs.	
		length of the root	increment per 1 h.	length of the root	increment per 1 h.	length of the root	increment per 1 h.	length of the root	increment per 1 h.
Water		5.6±0.4	0.233	12.8±1.8	0.267	5.4±0.3	0.075	5.6±0.4	0.058
Succinic acid	*M*/3000	8.0±0.7	0.333	22.0±2.7	0.458	7.1±0.5	0.098	8.6±0.7	0.090
Succinic acid	*M*/5000	8.2±0.8	0.342	14.5±2.1	0.302	6.0±0.5	0.083	7.1±0.8	0.074

The fact of stimulation of growth by substances formed as a result of a secondary alteration of amino-acids, particularly of such an amino-acid as aspartic acid, is beyond any doubt. It is of interest to note that it was found in the experiments of Koser, Wright and Dorfman ([4]) that an addition of aspartic acid in a quantity of 40 mg per 1 cm³ to the medium which contained no biotin caused a definite stimulation of the growth of *Torula cremoris*. Glutamic acid produced a lower effect and 17 other amino-acids had given no effect whatever. It is possible that the stimulating effect of the aspartic, glutamic and succinic acids is in some way connected with their dicarbonic nature.

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Comptes Rendus (Doklady) de l'Academié des Sciences de l'URSS
1945. Volume XLVIII, № 6

PLANT PHYSIOLOGY

EFFECT OF DAY-LENGTH UPON SALT-RESISTANCE IN COTTON

By V. A. NOVIKOV

(Communicated by A. A. Richter, Member of the Academy, 23. II. 1945)

The reduction of day-length is accompanied by a decrease in the total photosynthetic activity, the intensity of photosynthesis during the light period of the day remaining constant with the long or short day ([5]). Since photosynthesis is the main factor of yield, it is natural that, under equal conditions, the accumulation of dry mass is less with short than with long day ([3]).

According to the studies by Richter ([4]) the salt-resistance of the glucophytes, or at least their resistance to osmotic pressure produced by the salts in the soil, is dependent upon the osmotic pressure which is brought about in the plant's cells by organic substances.

As shown by the experiments of Blagoveshchensky ([1]) on potato, photosynthesis is sharply reduced by salinization. The total amount of dry matter as measured by the yield is also found to be reduced.

The grand total of the photosynthetic activity should, therefore, be reduced under conditions of salinization and of short day, both as a result of salinization and of the reduction in day-length. A higher susceptibility to salinization should, therefore, be expected with the short day-than with long one. If so, one should infer that the photosynthetic activity is a factor of salt-resistance.

The experiments on the effect of day length upon salt-resistance of cotton were carried on by me in 1941, at the Tashkent Agricultural Institute. Pots containing 20 kg of soil were used. The soil was fertilized as follows: per 20 kg of absolutely dry soil were added 2 g of nitrogen in the form of NH_4NO_3, 3 g P_2O_5 in the form of double superphosphate and 1 g of potassium in the form of KCl. No additional fertilizers were given in the course of the growing period. An early variety No. 2034 (*Gossypium hirsutum*) was subjected to experiments. Salinization to 0.2 *N* was carried out after the seedlings had appeared; for this purpose balanced Vant-Hoff—Richter's solution of chlorides was used. The salts were introduced into the soil at several doses with watering. The moisture content of the soil was kept throughout the growing period at about 60 per cent to the total moisture capacity. Short day (10 hours) was arranged in the corresponding experiments after the appearance of the seedlings. The replication was a threefold one. Each pot was made to contain a single plant. The data on the development of the plants are summarized in Table 1. One may see from the table that the rate of development with the short day regularly increased from butonization to flowering and from flowering to maturation; the same was observed in our previous paper on the effect of day-length on cotton ([3]). The early variety No. 2034 showed a noticeable acceleration with the short day: budding sets in by two days earlier, flowering—by four days earlier, and maturation—by eight days earlier. The reaction of the plants for the short day is greater with salinization than without it. Salinization

alone failed to produce any acceleration. This fact may be seen both in Table 1, and from the data of other studies ([2]). There is, on the contrary, even some retardation.

Table 1

Experimental sets	Dates				
	sowing	appearance of seedlings	beginning of budding	beginning of flowering	beginning of maturation
Natural day, without salinization	24.V	27.V	22 VI	20.VII	29.VIII
Natural day, salinization to 0.2 *N*	24.V	27.V	22.VI	20.VII	30.VIII
Short (10 hrs.) day, no salinization	24.V	27.V	20.VI	16.VII	21.VIII
Acceleration as compared with the natural day . .	—	—	2 days	4 days	8 days
Short (10 hrs.) day, salinization to 0.2 *N*	24.V	27.V	16.VI	12.VII	18.VIII
Acceleration as compared with natural day	—	—	6 days	8 days	11 days

A stronger reaction for the short day should be, therefore, considered as a combined effect of short day and salinization. Salinization was found to enhance the reaction of the plant for the short day, like in Čajlachjan's experiments ([7]) on *Perilla* and *Chrysanthemum* starvation and lack of carbohydrates made the plant more susceptible to short day.

The plants reared under the conditions of salinization (Table 2) produced smaller yields of total mass, of leaves, stems and raw cotton, especially with short day, and looked starving indeed as compared with those grown under the conditions of natural day. The yield of the valves, including both those of the mature bolls and of dropped bolls and flower buds, was almost the same with short day whether there was salinization or not. This should be explained by the fact that, against the background of salinization, the formation of flower buds and flowering was more intense than against the same background but without salinization; the behaviour was similar to that in Čajlachjan's experiments with starved plants exposed to short day. But since a great many of the flower buds and of young bolls were shed by the plants and those which reached maturity were considerably smaller than in the experiment without salinization, the yield of raw cotton (the yield of the valves being almost identical) proved to be with short day plus salinization one third of that obtained with short day, but without salinization.

Table 2

Experimental sets	Salinization	Yield of air-dry mass in gms and in per cent				
		leaves	stems	valves	raw cotton	total
Natural day .	0.0	21.6 100%	13.9 100%	12.8 100%	37.3 100%	85.6 100%
» »	0.2 *N*	17.3 80.1%	56.83 56.8%	5.3 41.4%	16.6 44.5%	47.1 55.0%
10 hrs. day .	0.0	15.4 71.3%	6.4 46.0%	3.8 29.7%	13.6 36.4%	39.2 45 8%
« »	0.2 *N*	8.2 38.0%	4.4 31.7%	3.4 26.6%	4.2 11.2%	20.2 23.6%

Judging by the data of Table 2, susceptibility of the cotton plant to salts increases sharply with the short day. The yield of air-dry mass with salinization to 0.2 *N* is reduced nearly to a half of the control under conditions of natural day, while with the short day it is less than one fourth of the control and one ninth of the yield with the natural day and without salinization. A reduced photosynthetic activity of the plant thus leads to a reduced ability to fight against the harmful effect of salts.

The determination of the content of chlorine (after Shestakov and Kachaev ([6])) in different organs of the plant (Table 3) was shown to be, as usual, higher with salinization, but short day which caused reduced salt-resistance did not produce any accumulation of salts either in the leaves or stems: only in the valves the chlorine content was found to be one and a half times greater than with natural day.

Table 3

Chloric Content in per cent of Air-dry Mass

Parts of plant	Natural day		10 hrs. day	
	no salinization	salinization	no salinization	salinization
Leaves	0.76	2.27	0.73	2.23
Stems	0.18	1.20	0.24	1.22
Valves	0.47	1.42	0.61	2.08

The harmful effect of similar amounts of salts is expressed to a different degree, being stronger with a lesser accumulation of the organic matter. It should be anticipated that the salt-resistance of the plant would grow along with the accumulation of organic mass.

Our studies permit of the following conclusions.

1. When cotton is cultivated under short day conditions its salt-resistance is reduced.
2. The susceptibility of cotton to the short day increases with salinization.
3. The increased reactivity of cotton to the short day under the conditions of salinization is connected with exhaustion.
4. The reduced salt-resistance of cotton with short day depends upon the reduced accumulation of organic substances within the plant.
5. It may be suggested that the salt-resistance of the plant will increase along with its photosynthetic activity.

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Received
23.II.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

EXPERIMENTAL MORPHOLOGY

ON THE NATURE OF THE INHIBITORY EFFECT OF LENS UPON LENS REGENERATION IN TRITON

By N. I. LASAREV

(Communicated by I. I. Schmalhausen, Member of the Academy, 10. XII. 1944)

Normally a single lens is developed in the eye of triton, which is preserved throughout the life. However, Sato (1) who transplanted the margin of the optic cup into the eye cavity instead of the lens removed showed that the eye cup and the margin of the optic cup are capable of developing an extra-lens. It is this ability which is evidently realized in the process of lens regeneration upon removal of the typical original lens. The question arises, why no regeneration of the lens takes place in the presence of the typical lens? And secondly, what is the nature of the inhibitory influence and the mechanism underlying this influence of the lens present?

There are but few investigations devoted to the problem of size correlation between the optic cup and the lens, bearing out the influence of the lens upon the eye (2) and the existence of a morphogenetic influence of the lens rudiment upon the eye rudiment (3). On the basis of these data it seemed possible to suggest that the inhibitory influence of the lens present upon the regeneration of the lens from the margin of the optic cup is morphogenetic in its nature. But if this suggestion is correct, then the development of an extra-lens must be suppressed in every case where the original lens is present in the eye. As a matter of fact, Sato described cases of simultaneous development of two lenses which arose as a result of regeneration from the margin of the optic cup and from an implant. Nikitenko (4) also observed the development of a lens in the posterior chamber of the eye from an implant when the original lens was present. These findings are evidence against the morphogenetic influence of the lens. The same objections can be put forward against the chemical nature of the influence here discussed. The only possible explanation is to assume a physical nature of the effect of the lens, which seems to be quite probable if the anatomical changes connected with the extirpation of the lens are taken into consideration. If the high intra-ocular pressure inherent in a normal eye be taken into account, which is demonstrated by the fact that the dissection of the cornea is immediately followed by a collapse of the eye, it is easy to imagine the pressure which the lens exerts upon the eye cup margins, extending them.

Upon the removal of the lens the margins of the eye cup join as closely as to leave sometimes but a very narrow pupil. As a result of the removal of the lens the margins of the eye cup are now exposed to the influence of the bottom of the eye cup.

Secondly, the free margin of the eye cup becomes swollen, which favours cell proliferation. These considerations served as a basis for the experiments described in the present paper.

The experimenal procedure adopted consisted in an immediate replacement of the eye lens removed by a paraffin globule of the same size. The paraffin was chosen because of its chemical and morphogenetic inactivity and easy handling. Adult *Triton taeniatus*, 13 in number, were used as experimental animals.

In vivo observations on the paraffin globules implanted showed them to be well fixed at the place of the lenses removed in 10 animals. They could be fairly well seen under a binocular microscope. A few days after the operation the eye seemed to be «quiet». A comparison between the experimental and the control animals showed neither traces of regeneration in the former nor regenerating lenses in the latter within a month after operation. By that time three control animals had lenses reaching half their normal size. Still larger lenses were present in the other two animals.

Microscopical examination of the animals with implanted paraffin globules confirmed the observations made on live material. It was found that regeneration of the lens did not take place if the implantation had been successful, and the paraffin globule implanted occupied the space of the lens removed.

These findings seemed to indicate that the suggestion as to the physical nature of the influence of the lens present was correct. However, no final conclusion could be ventured until a repeated removal of the implanted paraffin globule from the eye was carried out. If the paraffin globule was capable of «compensating» for the effect of the lens, then its removal from the eye must lead again to a regeneration of the lens from the margin of the eye cup Such an experiment was carried out. The removal of the paraffin globule was practised 27 days after the first operation. 6 days after the second operation the regenerating lens could be seen under a binocular microscope. The material was fixed on the 8th day and subjected to microscopic examination which confirmed the macroscopic *in vivo* observations. The regenerating lens corresponded to that usually observed at the 8-day stage of regeneration.

Therefore, the presence of the paraffin body in the place of the lens was quite sufficient to compensate for the influence of the lens removed.

When the present work had been ready for publication, there appeared a paper by Dinnean's ([5]), who implanted a newly excised lens at its proper place, and observed no regeneration of the lens. Therefore, the results obtained were similar to those of the first series of the present experiments. Dinnean pointed out that the implanted lens suffered a number of transformations characteristic of a foreign body rather than of a true lens. He arrived, however, to a premature conclusion as to the morphogenic nature of the influence of implanted lens inhibiting the process of lens regeneration from the margin of the eye cup. In the light of the present experiments the erroneousness of Dinnean's conclusion is quite evident.

It is important to point out that in two cases the implantations of the paraffin globules were unsuccessful because the globules broke to pieces. In these cases the implants were represented by paraffin fragments which were much smaller than the lens removed. In these cases the regeneration of the lens took place. This observation makes it possible to explain the regeneration in Sato's experiment in which the lens developed from the margin of the cup when the lens developing from the implant was present. The real cause of regeneration was that the implant was too small to prevent the formation of conditions necessary for the lens regeneration. Harrison observed an increase in the size of the cup transplanting into its cavity a lens that was characterized either by larger size, or a higher rate of growth. It is quite probable that these interrelations may be explained in terms of a mechanical influence of the lens. However, special investigation is necessary to verify this suggestion. Beckwich's ([6]) experiments, in which the absence of a lens caused underdevelopment of the eye cup, can also be accounted for from the point of view developed above. Naturally enough, the possibility of morphogenetic influences in the lens is by no means excluded. It is a well-known fact that the lens is capable

to induce the formation of the cornea ([7]). Nevertheless, it seems evident that the presence of the lens as a physical body plays an important rôle in the correlative interrelations of the constituent parts of the eye. We have become used to look for the sources of morphogenetic influence in every morphogenetic process. But there is no reason to search for influences morphogenetic in their nature that would suppress or inhibit some of these processes, since the suppressing or inhibitory influence upon the morphogenetic processes can be realized by any natural agent.

It is sufficient to say here that in spite of the presence of appropriate ability in the corresponding components, the lens fails to develop if the contact between the eye cup and the ectoderm is purposely broken.

This is a factor of physical nature, too.

The conclusion to be drawn from these experiments is that the regeneration of a lens does not take place under normal conditions, because of the presence of the lens as a physical body. The inhibitory influence in this case is due to an agent physical in its nature.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVIII, № 6

ZOOLOGY

A NEW ENDOPARASITIC MOLLUSC *PARENTEROXENOS DOGIELI*, NOV. GEN., NOV. SP.

By A. V. IVANOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 15. XII. 1944)

An interesting zoological finding was made by Professor V. A. Dogiel and Professor J. I. Polyansky in the surroundings of Vladivostok in autumn 1937. In the body cavity of holoturia *Cucumaria japonica* Semper they found long worm-like parasites which for their veliger-like larvae were identified as parasitic *Gastropoda*.

The material was handed over to me for investigation*. As shown by the analysis, the parasite is a new and very peculiar genus of the *Entoconchidae* family. Along with *Entoconcha* and *Enteroxenos* it belongs to the most regressive animal parasitic forms.

The young parasites, small or intermediate in size, are always attached by their posterior ends to the host intestine. Their anterior ends are lying in the body cavity of the host. The large individuals were found to lie free in the coelomic cavity of the host. As seen on the cross-section, the body of the mollusc is always covered from outside by the peritoneum and the underlying thin connective tissue layer of *Cucumaria*. A similar localization in the host body was described for *Enteroxenos östergreni* Bonnevie (1).

The body of the parasite is worm-like, elongated and rounded in cross-section. Both ends become gradually thinner and more or less spiral (Fig. 1). The same coiling tendency is known for *Entoconcha mirabilis* Joh. Müller (2). As the adult mollusc is many times as long as the host, it is necessarily forming numerous loops (Fig. 1) by which the internal organs of the host are entwined.

The length of the sex-mature individuals reaches 128.5 cm, the average thickness of the body is equal to 4—5 mm. The mollusc in question is, therefore, the longest form among all *Gastropoda* known, and nearly 9 times as long as *Enteroxenos östergreni* which was considered the biggest form of all *Entoconchidae* representatives described up to the present day.

The organization of the parasite is an extremely simple one. All its organ systems, excluding the sex system, have been completely reduced, as a result of its parasitic mode of life. Like *Enteroxenos* the mollusc here described is a hermaphroditic form. A considerable part of its body is occupied by a large pseudopallial or pseudomantlic cavity (Fig. 2, *C*, *psc*) which is quite similar to that of *Enteroxenos* and *Entoconcha*, and corresponds to the pseudopallial cavity of *Entocolax* and *Stiliferidae*. In big sex-mature individuals this cavity contains hundreds of cocoons with

* The author is very much indebted to Professor V. A. Dogiel and Professor J. I. Polyansky for having kindly put it at his disposal.

eggs or embryos. It does nearly reach the anterior body end and joins posteriorly with the cavity of the host's intestine through a narrow ciliary syphonal canal (Fig. 2, *C*, *s*). There is no perfectly developed syphon like in *Entocolax* ([3–5]) and *Gasterosyphon* ([6]). The wall of the hatching (pseudopallial) cavity plays, therefore, the part of a pseudomantle or pseudopallium, *i. e.* a large skin fold, stretching from the anterior part of the body. A comparison of the constitution scheme of the mollusc here described with that of other *Entoconchidae* (Fig. 2) goes to confirm the correctness of this interpretation. From the anterior wall of the hatching cavity stretches a large cylindrical, sausage-like outgrowth lying free in the cavity (Fig. 2, *C*, *vs*). The length of this outgrowth makes up one fourth of the total body length. It is probably the visceral sack whose exagerate development is connected with the progressive development of the ovary it encloses, while the increase in length is correlatively conditioned by the total lengthening of the body. The long and branched ovary (Fig. 2, *C*, *ov*) consists of an axial trunk and side branches. Anteriorly, the ovary is fused with the sex duct (Fig. 2, *C*, *ovd*) which nearly reaches the very anterior body end, bending backwards, and opening as a female sex aperture into the hatching chamber at the base of the visceral sack (Fig. 2, *C*, *go*). Spermatheca and other accessory organs are missing. Fertilization takes place in the proximal end of the sex duct, *i. e.* in the oviduct. A great amount of spermatozoa are seen there on the cross-sections.

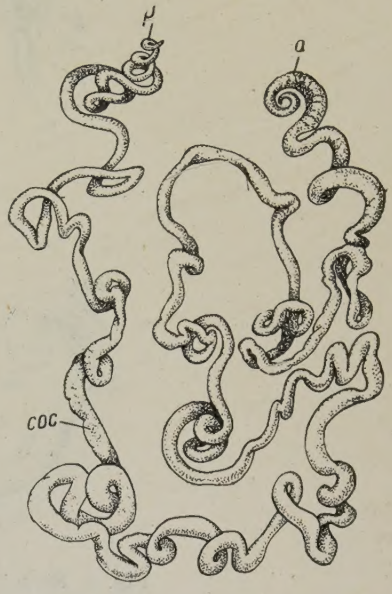

Fig. 1. *Parenteroxenos dogieli*. Sex mature individual. *a*, anterior end; *coc*, cocoons, seen through pseudopallium; *p*, posterior end

In the wall of the pseudopallial cavity near the syphonal canal is located a singular bean-like testis (Fig. 2, *C*, *t*); it lies under the epithelium of the hatching cavity, considerably protruding into the latter. The male sex duct is always poorly developed and mature spermatozoa penetrate into the hatching chamber through the wall of the testis. The testis is developed from a mesodermal rudiment which penetrates inside the epithelial fold of the hatching chamber from a mass of pseudopallium and is later completely partitioned from the connective tissue of the latter. In *Enteroxenos* ([1]) the development of the testes proceeds in a similar way. In the mollusc under consideration there is always present a single testis regular in shape. *Enteroxenos* has several testes; they are irregular in shape and connected with one another.

The pseudopallium of sex-mature individuals contains a large number (some hundreds or even thousands) of comparatively small cocoons with embryos and larvae (Fig. 2, *C*, *coc*). The larvae are about 0.1 mm long and have a shell which is bent in one plane like that of *Enteroxenos* and supplied with delicate and transparent operculum.

Thus, the form here described differs in a number of the essential characters from the rest of *Entoconchidae*, and should be considered as an independent genus for which I suggest the name of *Parenteroxenos*. The forms which are most closely related to it are *Enteroxenos* and *Entoconcha*. The latter resembles *Parenteroxenos* by its elongated body-shape, the presence of a big sausage-like visceral sack and its strongly developed musculature, differing from it by the presence of a syphon, the absence of an intestine, its hermaphroditism, and the shape of its shell.

Enteroxenos approaches *Parenteroxenos* with regard to its localization in the body of the host, the lack of intestine, hermaphroditism, the presence of a ciliary syphonal canal, and the shape of the larval shell, differing from it in the presence of a visceral sack, of strongly developed skin musculature, and a different construc-

tion of the sex apparatus. I call the new species *Parenteroxenos dogieli* in honour of Professor V. A. Dogiel.

Diagnosis of the genus. Body vermiform, strongly elongated. Oral aperture, intestine and all organ systems, except for the sex organs, are missing. Visceral sack well developed. secondarily increased and elongated, contains a long ovary. Pseudomantle separates from the anterior body end, surrounding the visceral sack and partitioning the large hatching chamber. Syphon represented

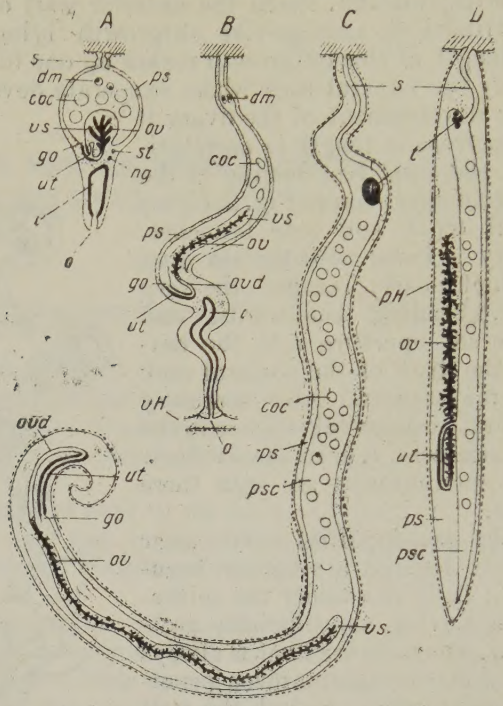

Fig. 2. Structure of *Entoconchidae* *A*, *Entocolax* ♀;*B*, *Entoconcha* ♀; *C*, *Parenteroxenos*; *D*, *Enteroxenos*; *coc*, cocoons with embryos; *dm*, dwarf males; *go*, female sex aperture; *i*, intestinal canal; *o*, oral opening; *ng*, nerve ganglion; *ov*, ovary; *ovd*, oviduct; *ps*, pseudopallium; *psc*, pseudopallial cavity; *s*, syphon; *s'*, statocyst; *t*, testis; *ut*, uterus; *vs*, visceral sack; *iH*, host's intestinal canal; *pH*, host's peritoneum; *vH*, host's blood vessel

by a narrow ciliar canal. The main trunk of the ovary bears branched and non anastomosing branches. Sex duct is bent, U-shaped and consists of a ciliar oviduct and a glandular uterus. The female sex aperture is situated at the base of the visceral sack. Receptaculum seminis missing. Hermaphrodite. A single testis lies in the posterior part of the body. Embryonic development takes place in the pseudopallial cavity into which the cocoons with the eggs come. The larvae supplied with the shell which is bent in one plane.

The single species—*Parenteroxenos dogieli* nov. sp., parasitic of the connective tissue layer of the intestine of holoturia *Cucumaria japonica* Semper, and reaches 128.5 cm in length. Place of finding: Japan Sea, Ussurian bay, 28.VIII. 1937 and 10.IX.1937.

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Received
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